CONFIGURATION OPTIMIZATION OF TRANSMISSION LINE TOWERS

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By

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To my Parents

KAMALAM and KARUPPAIAH

CERTIFICATE

This is to certify that the thesis entitled CONFIGURATION OPTIMIZATION OF TRANSMISSION LINE TOWERS by Mr. Karuppaiah Kumarasamy for the award of the Degree of Doctor of Philosophy is a record of bonafide research work carried out by him under my supervision, and has not been submitted elsewhere for a degree.

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November, 1979 K. KUMARASAMY

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A	area of cross section of the member
b	base width of tower, also distance from the edge of the fillet to the extreme fibre
C	matrix of damping coefficients
c_k	constant known from compatibility requirement
$\vec{d}$	vector of design variables $(d_1\ d_2\ \ \ d_n)$
DLF	dynamic load factor
E	modulus of elasticity
F	flexibility matrix
F_o	seismic zone factor, also load level at the starting of the regions considered (Wind load analysis)
F_a	allowable stress in angle sections
$f(\vec{d})$	objective function=total weight of the tower
$f(t)$	time function (max. ± 1)
g	acceleration due to gravity
$g(\vec{d})$	constraint function
h_i	height of the i-th panel from top in body
I	moment of inertia, also importance factor in earthquake analysis
K	structure stiffness matrix
k	b/t ratio, where t is the thickness of the angle
l	effective length of tower member
M	mass matrix(diagonal)
n_f	number of degrees of freedom
n_g	number of group of members in the tower

n_l	number of loading conditions
n_r	number of redundants
$\vec{p}$	vector of external loads
$p_i(t)$	load function at i-th degree of freedom $=F_i f_i(t)$
q_j	force in the redundant member j, also normal coordinate j in the M-orthonormal basis
r	radius of gyration
S_a	average ground acceleration
$\vec{s}_i$	search directions
$\vec{u}$	displacement vector
X_{k+1}	matrix listing the iteration vectors in the subspace
$y^{(u)}$	maximum allowable displacement Y
α_h	horizontal seismic coefficient
β	a coefficient depending upon the soil-foundation system; also $= \xi\omega$, where ξ is the fraction of critical damping
$\sigma^{(u)}$	maximum allowable stress σ
λ_i	i-th eigenvalue ($= \omega_i^2$)
$\lambda_i^{(l)}$	allowable lower limit of λ_i
$\lambda_i^{(u)}$	allowable upper limit of λ_i
τ	dummy time variable
ϕ_{ij}	normal mode of vibration of the tower, representing the deflection of the i-th mass in the j-th mode
ω	natural frequency of vibration
ω_j	undamped natural frequency of the j-th mode
ω_{jd}	damped frequency in the j-th mode
ρ	density of steel

SYNOPSIS

CONFIGURATION OPTIMIZATION OF TRANSMISSION LINE TOWERS

by

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With the enormous increase in the amount of electrical energy transmitted and the high voltage of transmission, the shape and size of the transmission line towers have received considerable attention in recent years. Since the tower structure is repetitive in nature, any saving in weight will build up the saving of material cumulatively. Hence the tower is most suited as an optimum design problem.

Sixties witnessed enough success in optimum structural design. Till early seventies these studies were by and large restricted to a fixed geometry of the structure. Off late attention is rightly focussed towards configuration optimization. In fact evolution of an optimal configuration should be the first step in any attempt of structural optimization. This thesis is directed towards the study of optimum configuration of Transmission Line Towers.

Automated optimum design of transmission line towers, modelled as a space truss, under static and dynamic behaviour is carried out with the purpose of generating optimum

configuration. The objective in the present work is to minimize the total weight of materials used including the weight of secondary members. The weight minimization is carried out subject to the limitations on static and dynamic stresses in the individual members and the requirement of compatibility of the tower configuration. The design variables chosen are the geometrical dimensions of the transmission line tower, viz., base width of tower and the panel heights in the body of the tower.

The justification to model the tower as a space truss both for static and dynamic analysis has been outlined in the present work. The decision to model the tower as space truss for dynamic analysis has been arrived from the study of eigenvalues of the tower. The configuration of the tower is generated automatically. Study of member forces under all possible independent loading conditions reveals that only three loading conditions are critical. The static analysis is carried out considering the maximum wind load as a static load. In dynamic analysis, wind load is considered time dependent. Dynamic analysis is also carried out for earthquake loading.

The optimum design problem is formulated as an unconstrained minimization problem. All the constraints are handled implicitly. This is necessitated since the

objective function is not an explicit function of design variables, viz., base width and panel heights. Powell's method has been used which turns out to be the obvious choice for seeking the solution of such unconstrained minimization problem.

A study has been conducted to assess the effectiveness of the two methods of analysis, viz., stiffness method and flexibility method, to be incorporated in optimum design. Stiffness method has been found to be efficient from computer time and storage requirements. The design of members of the tower is carried out as per the code IS: 802-1973, using available angle sections. From the areas picked up for the individual members, it is observed that certain angles are always left out during selection. This led to the conclusion that some of the available angles are not rational. From this observation a study has been conducted to bring forth a set of rational angles. These are presented in Appendix 2 of the thesis. Using these rational angle sections, the transmission line towers are again optimized for static loading which gives further reduction in weight.

Dynamic analysis of the transmission line tower is carried out through modal superposition. Lumped mass formulation is used in the present work. The resulting eigenvalue problem of the space tower is solved through

Subspace iteration method. The eigensolution is a series of bending modes and torsional modes in the lowest end of the spectrum. A parametric study has been carried out to study the variation of eigensolution with the height of the tower. From the results of parametric study, simplification on the modelling of slender towers for approximate dynamic analysis is recommended. The results of dynamic analysis, carried out separately for wind and earthquake loads, are tabulated.

The optimization study is carried out for 15m, 20m and 25m body heights of transmission line towers under static and time dependent loading cases. The effect of damping on the optimum weight of tower is also studied.

Some of the salient conclusions of the present study are: (1) the configuration corresponding to optimum tower is more sensitive to dynamic loads compared to the one obtained by considering only static behaviour; (2) for the design of transmission line towers, dynamic analysis under wind load should be carried out to have more realistic response prediction than considering equivalent static wind loads; (3) the conventional model of plane frame for the dynamic analysis of slender towers is confirmed to be reasonably accurate through the present study; (4) earthquake load is not critical for the design of transmission line

towers; and (5) available Indian Standard angle sections can be revised to more rational sections from the view point of minimum weight design.

CHAPTER 1

INTRODUCTION

1.1 General

The electric power generated at the thermal, hydro and nuclear power plants is distributed far and wide by a network of transmission lines. Transmission lines can, therefore, be compared to the circulatory system in the human body which distributes the energy required by the various parts of the body. Transmission lines are, as of today, a set of overhead conductors and a groundwire which transmit the electrical energy as high voltage current. Supporting structures are constructed at intervals to keep these lines at a clear height from the ground level. These structures are known as transmission poles or towers.

Transmission poles are used for relatively low voltage transmission (generally upto 132 kilovolts) and are generally made of wood, rolled or built up steel sections or prestressed concrete. Transmission towers are generally made of mild steel. Sometimes transmission towers of aluminum[1] and reinforced concrete[2] are used for transmission of voltages above 132 KV.

The transmission structures can be divided into two categories depending on their structural action, namely,

guyed or self-supporting. In guyed towers, guy wires carry most of the transverse loading and the central mast carries the vertical loading. A self supporting structure, as the name implies, carries the applied load all by itself.

The guyed tower[3,4] requires less structural steel but more right of way and often more construction effort. The tower can be completely assembled on the ground and erected as a whole with the help of a crane. The self supporting tower is heavier, but has the advantage of occupying less ground area and providing greater stability. Guy anchors[5] require excellent compaction to avoid yielding and slackening of the guys, whereas the average self supporting towers can tolerate a certain amount of substandard backfilling of its footings. Also, the guyed towers are vulnerable to sabotaging. Self supporting towers are, therefore, more widely used than the guyed towers. Hence the present study is directed towards the self supporting transmission towers of the kind shown in Fig. 1.1.

Transmission towers are divided into three main categories depending on their location. These are:

(a) Tangent or suspension towers (Fig.1.1): These are used in the straight sections of the lines. They are designed to support the wires when subjected to wind and ice loads and frequently are designed

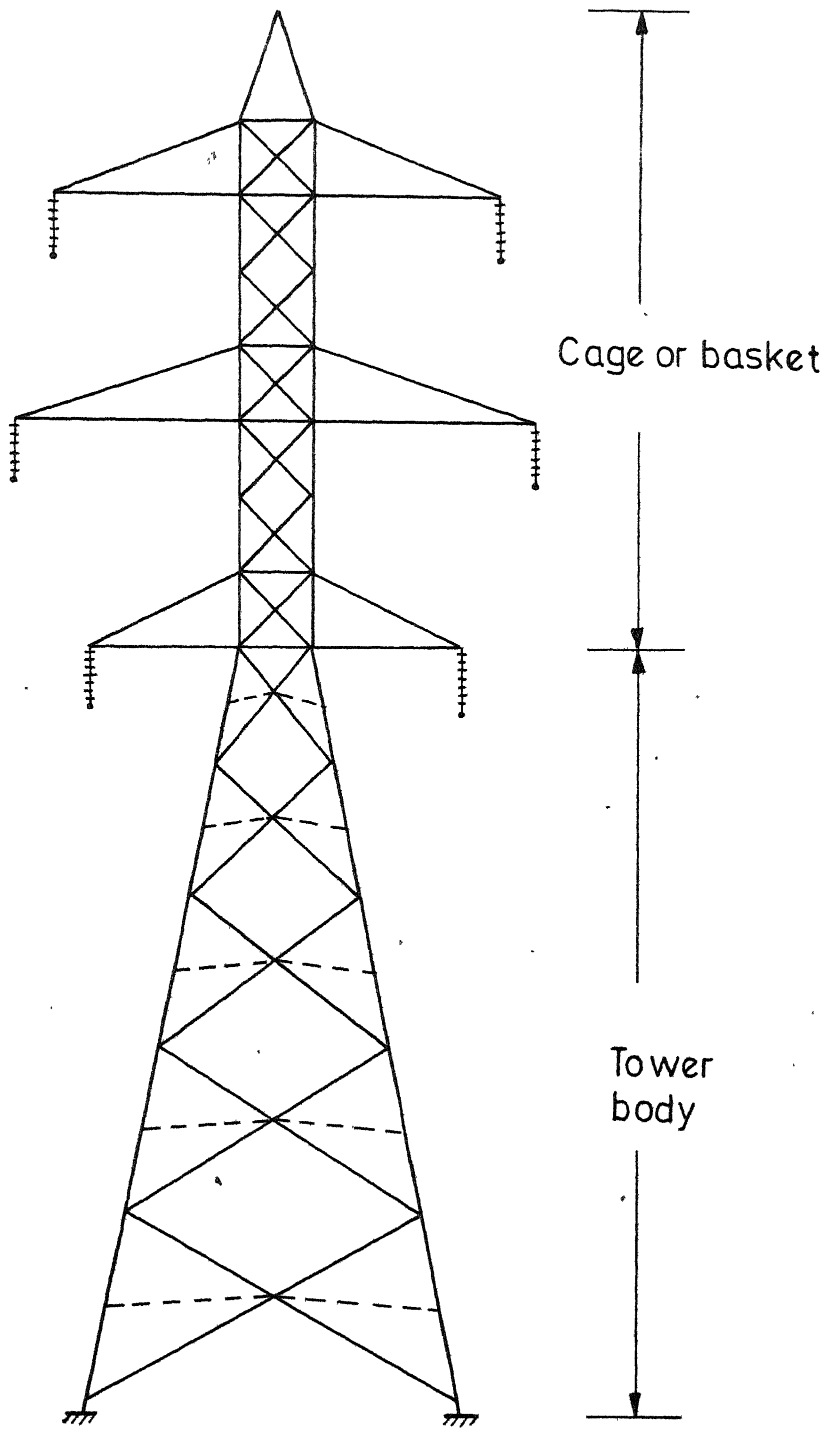


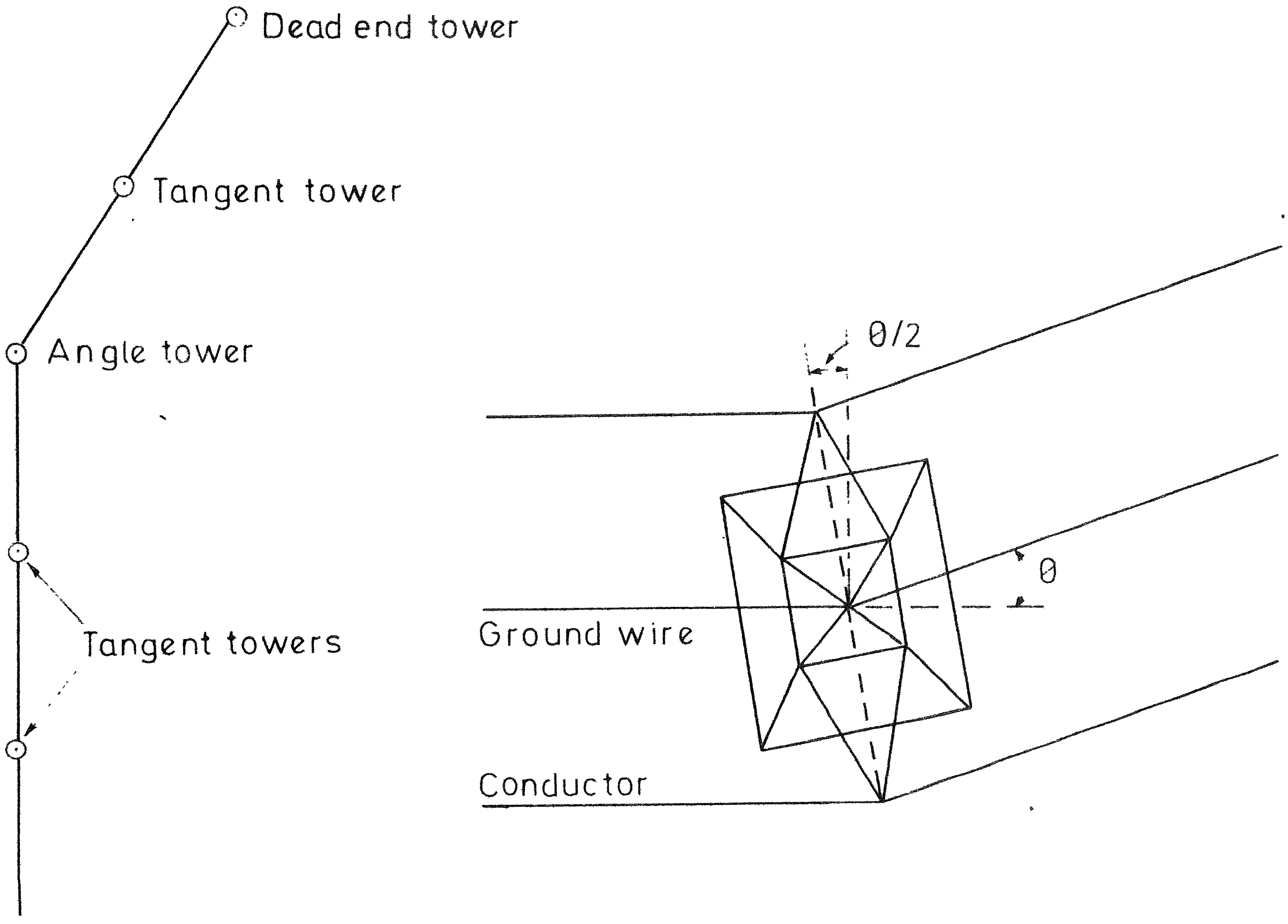
Fig. 1·1 Typical double circuit transmission line tower

for a small angle change in the line. They are also designed for the unbalanced pull due to a broken wire on one side of the tower.

(b) Angle towers: These are located at points where the line changes direction. They are designed to carry the same load that are carried by the tangent tower in addition to a transverse pull due to the change in the line direction. In order to utilise the structure most efficiently, the structures are usually placed so that one of the axes of the cross section bisects the angle formed by the conductors as shown in Fig.1.2.

(c) Dead end or anchor towers: These are designed to take the dead end pulls from all the wires on one side, together with wind and vertical loads.

The trend in the electrical transmission technology is towards the use of very high voltage for transmission. Voltage of the order of 750 KV[6,7] are already popular in the western countries. Dillard and Hileman[8] have reported that transmission of 1100 KV is technically feasible. This increases the load on the structure and also the clearance requirements and consequently larger structures are required. An accurate analysis of the tower under these heavy loading and a synthesis of the analysis and design procedure to



(a) Location of different categories of towers

(b) Orientation of an angle tower

Fig.1·2 Classification of towers based on location

economise the cost of the towers is warranted. The computerised analysis and optimum design is the natural choice of the means to achieve this end.

1.2 Review of Literature

The first known paper on transmission line towers is by Bergstron, Arena and Kramer[9]. This is the state of art type paper, describing the salient features of the design of self supporting transmission towers subject to the then existing code of practice—the National Electric Safety Code, the shortcomings of this code and the necessity to modify the code. The main points suggested in the paper are as follows:

1. Selection of design loading conditions comparable with the anticipated loads rather than provision for every remote contingency.
2. Willingness to consider rectangular tower cross section with its increased complexities.
3. Extensive studies on various configuration to achieve optimum strength with least material.
4. Careful checking and rechecking of members to ensure selection of minimum sizes and the use of a computer program to select members automatically.
5. Incorporation of high strength steel in the design at advantageous points.

The method of least work has been used in this paper for the analysis of the tower as a space truss.

Selection of site and heights of towers, depending upon the terrain, is a major factor in the design of transmission system. Dynamic programming or recursive optimization is especially useful for multistage process in which decisions taken at one stage do not affect the previous stages. Dynamic programming approach has been successfully employed[10-12] to design the transmission line tower location along the route so that the construction costs are minimized, while the physical constraints are satisfied. The input data required include the survey data of the route and the choice of available towers of suspension type and of angle towers.

Having fixed the tower location along the route of the line, the next job is to decide the outline diagram, length of crossarm and conductor spacings, tower width at the base and top hamper, and type of bracing pattern[13]. Gopalan[14] has presented the procedure for the design of barrel towers and shown the transverse and longitudinal faces of 132 KV double-circuit barrel type tower. The paper by Lo, et al[15] describes the computer program developed for the analysis and design of steel transmission towers or general truss. The program capabilities consists

of (1) generating symmetric portions of geometry; (2) identifying and stabilizing planar (unbraced) joints; (3) handling only tension members; (4) accounting for the buckling of members by using a bilinear force-displacement relationship; and (5) varying some key joints to optimize the tower geometry.

In the western countries, computers have been used to design completly an entire stretch of transmission line, which include the selection of conductor type, layout of the towers along the line, fixing the dimensions of the towers to satisfy clearence requirements, design of the towers and inventory control of structural and electrical components. An excellent account of an integrated transmission line system design developed by Ms. Sargent and Lundy Engineers is given by Beck[16]. The fundamentals involved in the design of a transmission tower have been described vividly by Marjerrison[17].

With the introduction of extra high voltage (EHV) for transmission of electrical power, and the availability of the digital computers for analysis and design, a rational assessment of loads, allowable stresses and overload factors have become necessary and this was clearly brought out by Farr, et al[18]. A review of past practices in the calculation of loads on transmission line towers, as used in the design,

is given and a new concept in design is suggested using more realistic ultimate load approach. Campbell[19] has presented a non-linear analysis of an elastic transmission line for conditions involving imbalance in conductor tensions. Stiffness properties of structures, conductors and insulators are derived considering structure and insulator displacements.

Early design of transmission line towers were carried out by analysing the approximate determinate structure either by resolving the joint forces or by graphical methods. Today, with the advent of high speed digital computers, it is possible to analyse the entire tower as an indeterminate structure. Natarajan[20] has presented a simpler version of the frontal solution technique suitable for structural analysis with large number of members. The method has been used to analyse a transmission tower and the resulting stress distribution in the main members are discussed. The elastic transmission system power flow equations bear a striking similarity to force-displacement characteristics of pin-joint structures such as electrical transmission towers. This similarity is successfully exploited in order to radically increase the speed of analysis of transmission line structures[21].

In general transmission line towers are tested, under the expected loads, so as to guarantee the safety of the system under working conditions. Murthy and Mukerjee[22] have reviewed the methods and practices followed in some of the advanced countries as well as in India as regards to the testing of transmission line towers. A uniform procedure for testing the steel transmission towers is presented with suggestions for the improvement of the testing practices.

The paper by Anaston[23] describes a computer program written to design optimally a complete transmission tower of given basic dimensions. The tower, divided into five parts, has been analysed as a statically determinate system and the members are designed according to code specifications. The variables involved in the procedure are the width of the basket, width of the base, number of panels in the basket and number of panels in the body. The input consists of basic dimensions and loads. The configuration has been generated by the computer and the analysis and design are performed. The output consists of the values of the variables mentioned above, and the sizes and lengths of all the members. A distinguishing feature in this study is that the weight of the tower, excluding the weight of secondary members but including that of anchors, is minimized. The paper, however,

does not mention how the variables were varied and the amount of variation in each variable.

Raj and Durrant[24] have presented dynamic programming formulation for structural design optimization in general and discussed possible extension to transmission line tower configuration.

Verma[25] has analysed basket portion and body of the tower separately and developed a program, using force method of structural analysis, for minimum weight design of transmission line tower with square base. The design variables were panel heights of the body and base width of the tower. Using Univariate method of unconstrained minimization, he concluded that panel heights have considerable effect on the minimum weight of transmission line towers. Kuzamanović, et al[26] have reported considerable weight reduction by using three legs instead of classical square base for the transmission line towers.

No effort seem to have been put, so far, towards the development of rational optimum design of transmission line towers subject to both static and time dependent loading.

1.2.1 Optimization of Truss-Like Structures

Michell[27] has solved the following problem of structural optimization: Given an equilibrium system of

forces (loads and reactions), to design a plane truss with bars of a given elastic material in which the given forces produce axial stresses within a given allowable range, with the amount of material required for the bars is as small as possible. Michell's optimal structures, however, are not trusses in the customary sense of the term but truss-like continua with an infinity of joints. Structural optimization problem today is nothing different from that of Michell's , but stated in the generalised form. The problem statement varies only depending upon the objective of the problem.

Excellent surveys by Wasiutynski and Brandt[28] in 1963 and Shen and Prager[29] in 1968 provide a complete historical development of the analytical methods of structural optimization. Mathematical programming techniques applied to structural optimization problems were reviewed by Kapoor and Hariharan[30] till 1972. Recently Rozvany and Mroz[31,32] have reviewed the progress in structural optimization at a conceptual level.

Early works on structural synthesis were applied to truss-like structures, being simplest of structural forms, to arrive at a minimum weight. The attention was focussed to get minimum cross section for the members of the truss subject to non-zero value of cross section, stress and

local instability constraints[33]. This resulted in a set of necessary and sufficient conditions for local optimality of a fully stressed design under a single system of loads[34]. Lev[35] has presented an algorithm that starts with an indeterminate truss and at each cycle of the procedure one bar (or more) is eliminated, thus yielding an optimal truss with a lower degree of indeterminacy, until the final solution is reached. Based on the least-weight indeterminate and least-weight determinate truss, useful bounds on the truss weight has been obtained which can be used as a criteria for efficient design. Prager[36] considered the near optimal trusses. The upper and lower bound optimal layout of truss was discussed that requires an amount of material exceeding the minimum possible amount by at most a known percentage.

With the initial success in minimum weight design of simple structures, the question arose whether minimum weight design and fully stressed design are the same. Dayaratnam and Patnaik[37] have reported that the fully stressed design of a statically indeterminate truss is not feasible for single load condition. There exists a relationship between the number of load conditions and the order of indeterminacy of the structure for a feasible fully stressed design.

The general structural optimization problem is to get the minimum of the objective function under a set of constraint functions which are, in general, nonlinear. These nonlinear functions of constraints and objective function can be expanded in Taylor's series about the initial design point. If the series is approximated by retaining only the first two terms (constant and linear term), then the problem becomes a linear programming problem which is comparatively easier to solve. The solution can be obtained in the direct form or in the dual form[38] depending on the computational effort required. In general this type of transformed SLP (sequence of linear program) problems produce unacceptable designs. To eliminate infeasible designs, a set of pseudo constraints[39] are added to the original linearized constraints. This way the problem size becomes large which requires more computational effort and slow convergence to the optimum of the original problem.

Based on the analysis of direct (mathematical programming) and optimality criteria (indirect search) methods for structural optimization, efficient hybrid methods were suggested exploiting common features of both methods. Fleury and Geradin[40] have shown, through numerical applications, significant improvement in convergence properties with hybrid method. Arora and Haug[41] have discussed the application of

hybrid methods to dynamic response problems.

With the success in application of mathematical programming methods to statically loaded structural optimization problems, attempts were made to optimize structures under dynamic and stability constraints. Fox and Kapoor[42] have optimized the truss-frame structures, using the method of feasible directions, under dynamic response regime. The members were designed under peak stresses. Cassis and Schmit[43] have used the first-order Taylor series expansion of the dynamic response functions with respect to the design variables. The Davidon-Fletcher-Powell algorithm has been used for the unconstrained minimization.

Khot, et al[44] considered the minimum weight design problem with stability constraints. The method was programmed for trusses and frames. Schmit and Ramanathan[45] have reported the minimum weight structural design of truss and wing structures in a multilevel approach. The overall proportioning of the structure was achieved at the system level subject to strength, displacement and system buckling constraints while the detailed component designs were carried out separately at the component level satisfying local buckling constraints. The system buckling behaviour was handled by incorporating the geometric stiffness matrix capability.

Schmit and Miura[46,47] have developed programmes that combines finite element analysis techniques and mathematical programming algorithm using an innovative collection of approximation concepts. Design variable linking, constraint deletion technique, and approximate analysis methods were used. Their earlier paper[46] discusses the application of ACCESS-1 (Approximation Concept Code for Efficient Structural Synthesis) to systems represented by truss, constant strain triangle and symmetric shear panel finite elements. The later advanced version ACCESS-2[47] includes problems involving fibre composite structure, thermal effects and natural frequency constraints in addition to the usual static stress and displacement limitations.

Till early seventies, studies on structural optimization were, by and large, restricted to a fixed geometry of the structure. Of ~~f~~ late attention is rightly focussed towards configuration optimization. In fact evoluation of an optimal configuration should be the first step in any attempt of structural optimization.

Vanderplaats and Moses[48] have reported significant weight reduction when geometric changes are included in the optimization process. The member areas and joint coordinates were treated as design variables. The design

problem was, in effect, divided into two separate, but dependent, desigr spaces— one for member sizes and one for coordinates. While changing coordinate variables the member areas were treated as dependent variables. Fully stressed design was assumed optimal. Kunar and Chan[49] have formulated the minimum weight design of structures with variable geometry. The member sizes were expressed in terms of configurational variables satisfying the imposed constraints by use of optimality criteria. The problem was solved via an unconstrained minimization algorithm.

Bennett[50] has presented a two step procedure in which the first step include only the cross section variables and for the second step both cross section and layout variables were used. Method of feasible directions has been used for the minimization. Illustrative examples include frames. Lipson and Agarwal[51] have applied complex method of Box to optimize planar truss structures. The design variables included geometry and member area using the available sections. This method has been extended successfully to space trusses[52,53] and comparison was made with minimum weight design obtained with continuous variables. Favourable design improvements and rates of convergence were demonstrated.

1.3 The Present Work

The computational capability for the automated optimum design of transmission line tower modelled as a space truss under linear elastic static, and dynamic behaviour is developed in the present work with the purpose of generating optimum configuration. The objective function chosen is the total weight of materials used including the weight of secondary members. The weight minimization is carried out subject to the limitations on static , and dynamic stresses in the individual members and the requirement of compatibility of the tower configuration. The design variables chosen are the goemetrical dimensions of the transmission tower, viz., base width of tower and the panel heights of body of the tower.

The necessity of modeling the transmission tower as a space truss structure both for static and dynamic analysis is brought out in Chapter 2. Automated generation of tower configuration is the first step towards the study of configuration optimization. This is achieved in two parts. The first part generates the fixed data such as the joint connectivity of members, and type of members and this is done only once during the whole optimization routine. This subroutine also generates the coordinates of the joints in the basket portion, as this part of the tower configuration

is fixed. The second part generates the variable data such as the coordinates of joints in the body of the tower and length of secondary members, which get modified at each design point. This is described in Chapter 2. Transmission line tower is to be analysed for a set of load conditions as per IS:802-1967[54]. These are discussed in Chapter 2. Out of all possible conditions only a few turn out to be critical load conditions from the view point of design. These are brought out in Chapter 2. In the 1973 revision[55] of IS:802, the wind pressure values have been increased by 30 percent to account for dynamic effect, irrespective of the height of tower. The constant increase in wind pressure for all heights does not seem to be correct. Any measure to account for the dynamic effect has to bring-in the stiffness of the structure. This is important while designing slender structures like transmission line towers. Hence a deterministic dynamic load has been approximated wherein the maximum wind pressure at various heights correspond to those values given in IS:802-1967. This is discussed in Chapter 2. Earthquake loads for the design of transmission line tower are also discussed in this chapter.

Chapter 3 deals with the formulation and method of optimization of the resulting optimum configuration

design problem. The implicit nature of the objective function has made it necessary to formulate it as an unconstrained minimization problem. Powell's method turns out to be the obvious choice for seeking the solution of such an unconstrained minimization problem.

Structural analysis consumes a major part of computer time in the design of transmission line tower. Optimization process being a repetitive analysis design cycle, it is of utmost importance to use an analysis technique which consumes less time. Hence for the present work both flexibility and stiffness methods of structural analysis are studied to assess the effectiveness of one over the other from the computational point of view. Stiffness method has been found to be efficient from computer time and storage requirements. This is discussed in Chapter 4. The design of members of the transmission tower is carried out as per IS:802-1973[55] using available angle sections[56]. The area and other sectional properties of available sections are stored in the memory of the computer in ascending order of their areas. The results of optimization of the tower configuration under static loading is presented in Chapter 4. From the areas picked up for the individual members, it is observed that certain angles are always left out during selection. This led to the conclusion that

some of the available angles are not rational. From this observation a study has been conducted to bring forth a set of rational angles. These are proposed and presented in Appendix 2. Using these rational angle sections, the transmission line towers are, again, optimized for static loading which gives further reduction in weight. The details are presented in Chapter 4.

Chapter 5 deals with dynamic analysis of the transmission line tower. Modal superposition method, used in the present work, is briefly discussed in this chapter. The resulting eigenvalue problem is solved using subspace iteration method. A parametric study on eigenvalues by varying the height of tower is carried out. The results of dynamic analysis for wind and earthquake loads are presented separately. The optimum design under static loading condition, obtained in Chapter 4, is taken as starting design for carrying out optimization study under dynamic loads. These results are also presented in Chapter 5.

The conclusions drawn from the present study and suggestions for further extension of the present work are presented in Chapter 6.

CHAPTER 2

TRANSMISSION LINE TOWER STRUCTURE

2.1 Introduction

The transmission line tower consists of two parts (1) the basket or cage, and (2) body as shown in Fig. 1.1.

The basket portion of the tower has the cross arms projecting from the tower shaft which in turn support the suspended conductors. The vertical spacing of these cross arms mainly depends upon insultation requirements. This portion of the tower is detailed from electrical considerations.

The body of the tower extends from the lowest cross arm upto the support. The height of body of the tower may vary from tower to tower depending upon the local geographical conditions. The configuration is decided by the width of base, number of panels and height of each panel.

The tower structure consists of one dimensional slender members, as is seen from Fig. 1.1. These members can be classified as leg members, horizontals and bracings. They are generally rivetted at ends to form joints. Sometimes the joints are also bolted connections. These type of connections lead to partially fixed condition at the joints. However, the slender nature of the members of the tower make

the effect of partial fixity of the joints negligible. Hence the joints of the tower are assumed to be hinged for the purposes of analysis. This idealization results in axial forces of tension or compression in the members when the tower is subjected to joint loads.

Having identified the structure, deciding a suitable model for analysing the tower structure, subject to static and dynamic loading, is the first step in the present study. This is discussed in Section 2.2. Automated generation of tower configuration as well as design load conditions are also discussed in the subsequent sections.

2.2 Modeling of the Tower Structure

In the design offices, the analysis of the tower structure is oversimplified. The simplification is two-fold: first, the tower which in fact is a space structure, is idealized to a planar truss and secondly the analysis of planar truss does not take into account the presence of redundants. In other words planar truss is analysed as a determinate structure. These simplifications are not justificable for the following reasons:

(1) Projected dimensions of the tower, on a vertical plane, are considered.

(2) Loading on the tower, particularly the situations in which one of the conductors is broken, is not planar.

Moreover, the simplifications discussed above which were primarily due to computational ease are no more warranted with the availability of high speed digital computers.

A study has been conducted by Sondth and Mukhopadhyay[57], modeling the tower as planar as well as space truss subjected to static loads. The computerised results obtained for the member forces show that, in general, the forces are overestimated in the planar model. Moreover, some members also seem to be highly understressed. Hence the tower structure has to be modelled as a space truss.

2.2.1 Model for Static Analysis

Modeling the transmission line tower as space truss result in three degrees of freedom at each joint, one each along the coordinate axes. The base support is assumed fixed for translations. Hence the number of degrees of freedom for a transmission tower with n joints(excluding supports) is 3n.

2.2.2 Model for Dynamic Analysis

The transmission line towers vibrate transversely in the bending modes, when all wires are intact. However,

when ground wire is broken the tower vibrates in bending in the longitudinal direction of the line. Furthermore, when one of the conductors is broken, the tower vibrates in torsional modes. These considerations make it necessary that the transmission tower should be modelled as a space frame.

The main reason to model the structure as frame is that the individual member frequency of long member, assumed as simply supported at ends, may have fundamental natural frequency less than that of the structure[58]. In such a situation that particular member may vibrate with high amplitude and lead to failure which in effect mean collapse of the structure.

The members of the transmission line tower are very slender. Hence the rotational resistance offered by the members at the joints are negligible. Also the overall structure is slender. So the stiffness of the overall tower both for space truss and frame model remain close. Hence as a first step, the tower is modelled as a space truss for the eigenvalue analysis.

The tower studied has 6 panels in the body and the base width of the tower is taken as 4.33m, shown in Fig. 2.1. Lumped mass formulation is used, Fig. 2.2. First 6 natural frequencies (square-root of eigenvalues) of the tower are

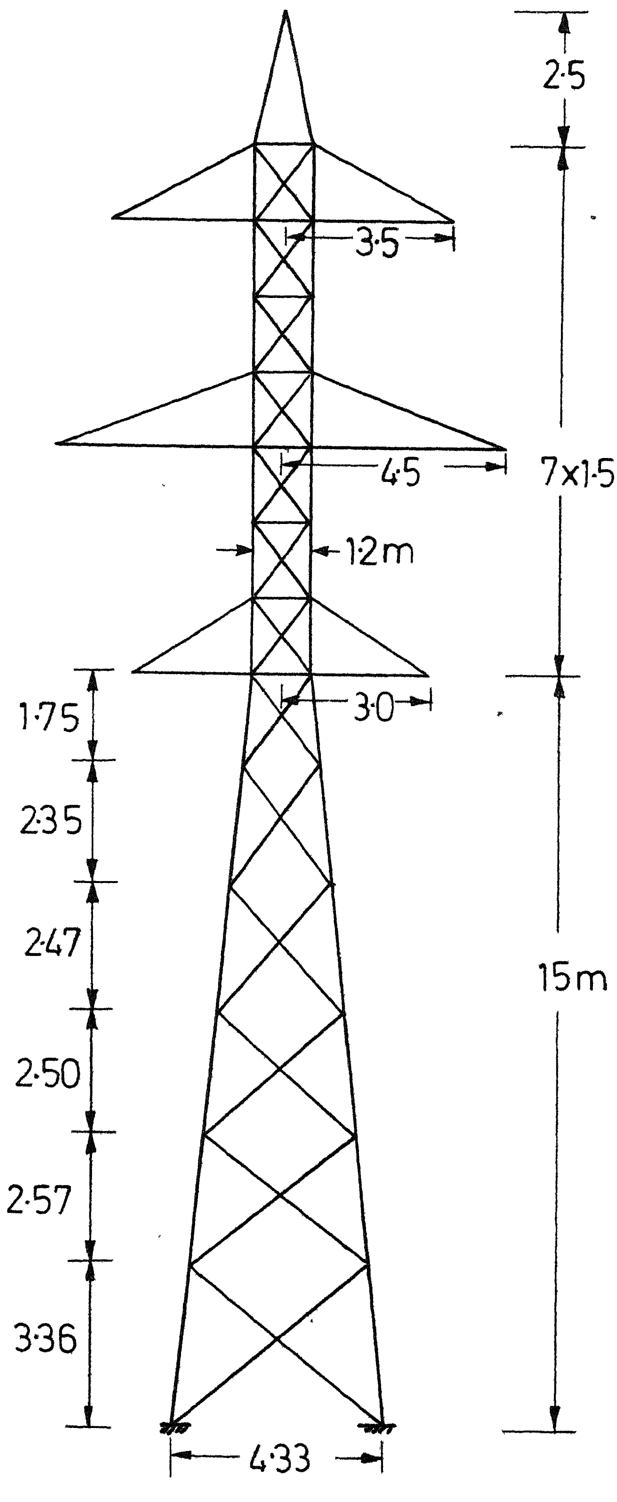


Fig.2·1 Tower analysed for eigenvalues

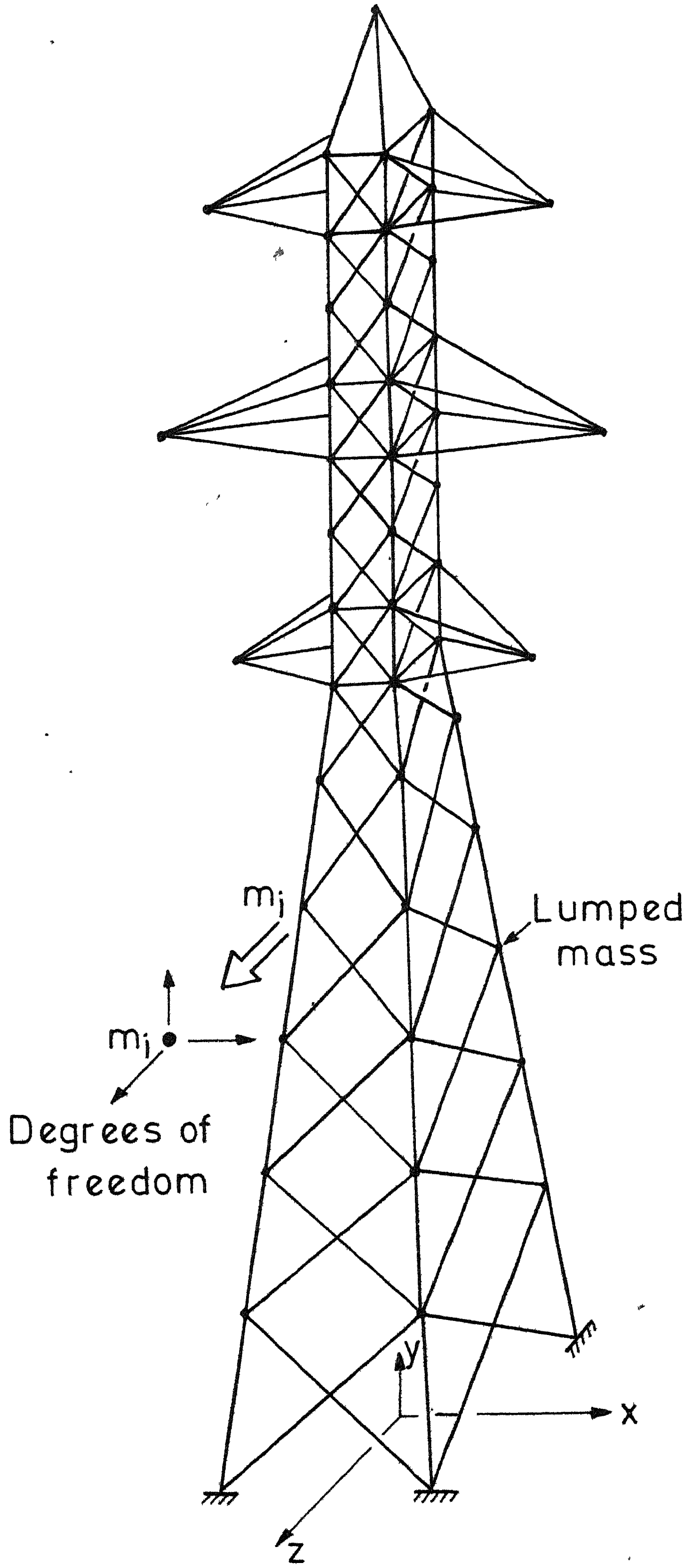


Fig.2·2 Tower idealization

extracted using subspace iteration method. The results are presented in Table 2.1. The first eigenmode is the bending mode in the transverse direction, second is the bending mode in the longitudinal direction of the line and the third is the torsional mode. The fourth, fifth and sixth natural frequencies are very well separated from the first three. This shows that the major portion of response mainly depends on the first three modes.

Table 2.1 Natural Frequencies of Transmission Tower with 15m Body Height

Mode No.	1	2	3	4	5	6
Structure frequency (rad/sec)	26.3	26.6	35.1	77.7	84.0	89.6

Coming to the individual members, the long member is the diagonal in the bottom most panel. The length of this member is about 5.25m. Considering this member as simply supported, the lowest natural frequency is given by

$$\omega_1 = \frac{\pi^2}{l^2}\sqrt{\frac{EI}{m}} = \frac{\pi^2}{l}\,\frac{1}{(l/r)}\sqrt{\frac{EA}{m}} \qquad (2.1)$$

where $m = \dfrac{\rho A}{g}$

Substituting the values, $\rho = 0.00783$ kgf/cm^3, $E = 2.047 \times 10^6$ kgf/cm^2, $g = 981$ cm/sec^2 and the maximum value[55] of (l/r) = 200, we get

$$\omega_1 = 47.75 \text{ rad/sec}$$

Since this diagonal is subdivided by the other diagonal in the same panel (also by secondary braces), on the face of the tower, the lowest natural frequency become $\omega_1 = 4 \times 47.75 = 191$ rad/sec. But in the perpendicular plane to the face of the tower, however, the member frequency is only 47.75 rad/sec. Hence the lowest natural frequency of the long member is well above the first three natural frequencies of the tower. Therefore, the response of the overall structure shall govern the design rather than the response of individual members.

It is concluded from the above discussion that the transmission tower need be modelled as a space truss. The space truss model result in the number of degrees of freedom same as that in static analysis.

2.3 Configuration

2.3.1 Basic Bracing Configuration:

There are three types of bracing systems which have been mainly used on tower bodies (Fig. 2.3).

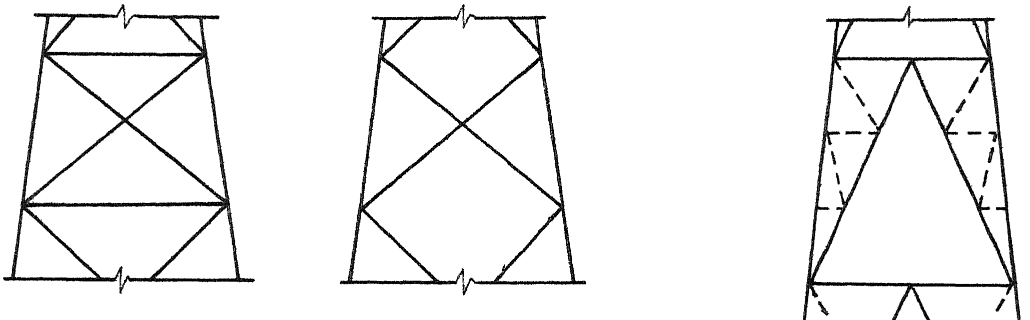
(a) Tension system,

(b) X-braced tension-compression system, and

(c) K-braced tension-compression system.

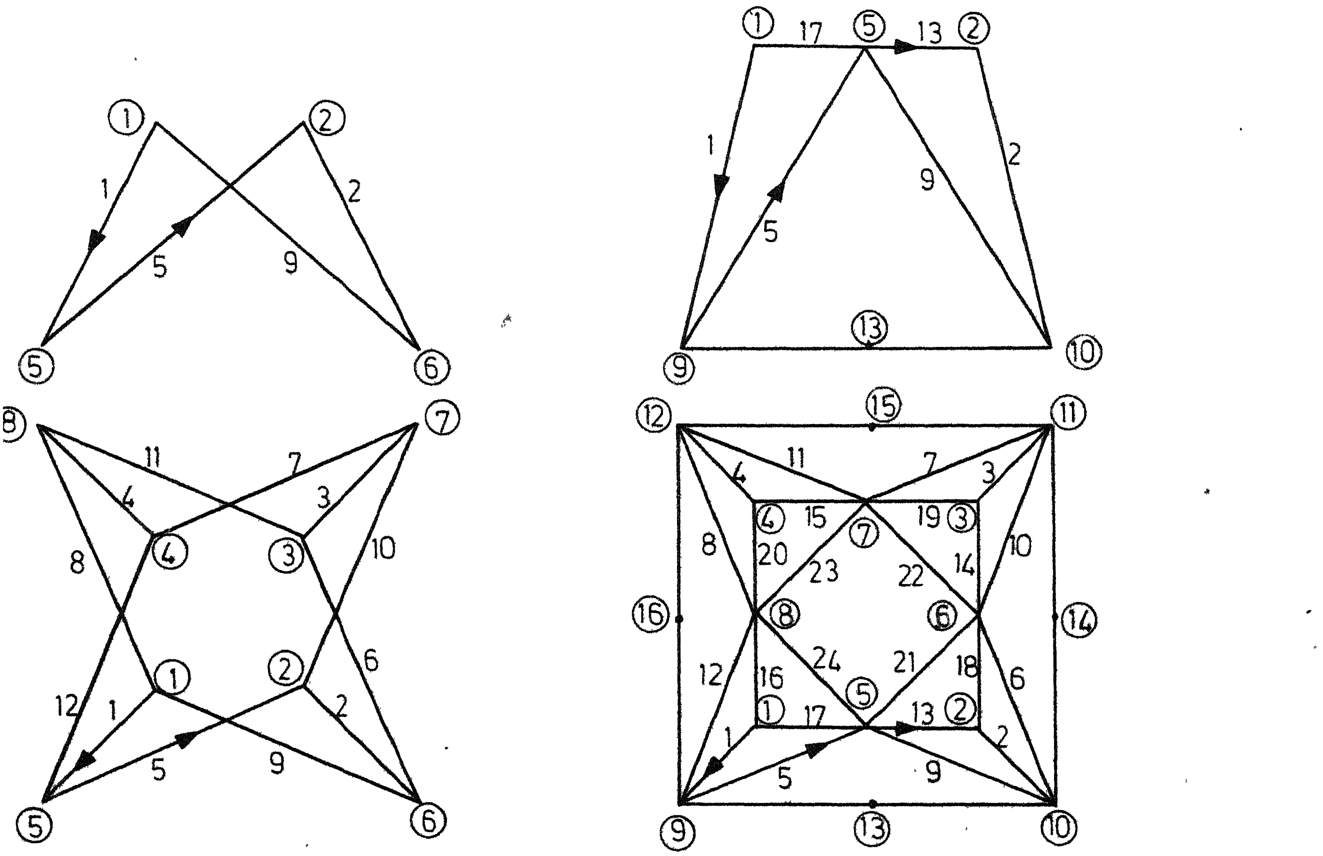
In the tension bracing, all the diagonal members have slenderness ratio high enough to act only in tension. Such bracing results in large deflection under heavy loading, because the tension members are smaller in cross section than what the compression members would be for the same loading. Overloading of such towers would result into the failure of the inactive tension member in compression, and the active tension members would carry the load and prevent the collapse of the tower. X-type of bracing arrangement is very efficient for relatively small lateral dimension of the tower. Such arrangement gives better distribution of load to the tower footing. K-type is generally applicable to large structures and has not been found competitive with either the tension system or the X-bracing for normal suspension towers.

It is possible, in general, to combine two or more of the above basic types to obtain a combined bracing configuration for a particular tower, which may prove to be economical. However, for the purpose of this study, it has been assumed that all the panels have X-type of bracing which has been found to be economical for the body heights upto 25m[59].



(a) Tension system (b) X-bracing (c) K-bracing

Fig. 2·3 Basic types of bracing



X-type K-type

Fig.2·4 Joint and member numbering for configuratior

2.3.2 Generation of Configuration

The configuration of the tower is generated in two parts. The first part generates the fixed data such as connectivity of the members and type of members, which are computed only once during the optimization cycle. The second part generates the coordinates of the joints and lengths of secondary members which are generated after any change in the design variables.

2.3.2.1 Generation of Fixed Data: Subroutine 'DATAIN'

The program has been written in general so that data for any latticed tower structure can be automatically generated with the minimum possible information as input. The tower structure has joints interconnected by members, located on horizontal planes at various levels. At such a plane lies four or eight joints as shown in Fig. 2.4 depending upon the type of bracing adopted, i.e., X-type or tension bracing result in 4 joints in a horizontal plane while K-braced panels result into 8 joints. These joints are located equidistant from the vertical axis of the tower and hence if the coordinates of one joint (henceforth called as reference joint) is known, then coordinates of the other 3, in the case of X-bracing, can be generated. The joints are numbered sequentially in the anticlockwise direction, starting from left joint on the front face of the tower, in each level.

For example, the coordinates of (1) in Fig. 2.4 will only be fed and the coordinates of (2), (3) and (4) will be generated simply by changing the sign of the coordinates of (1). If a panel has K-bracing, then the coordinates of the middle joint on the front face of the tower is also required to be fed.

The other information required for the tower configuration is the connectivity table of the members. For this purpose the members of the tower are grouped as leg members, diagonal members, horizontals etc., in each panel. To generate the connectivity table, the connectivity of only one member from each group is read and for all the other members, the connectivity is generated based on symmetry of the tower. This is possible by suitably numbering the joints as shown in Fig. 2.4. For example, connectivity of leg member 1 will be fed as data while the same for other members 2, 3 and 4 is generated by suitably incrementing the joint numbers of member 1. Similarly for diagonal bracings,connectivity for member 5 is fed while the data for other 7 members in this group are generated. In Fig. 2.4, the members are numbered starting from 1 for the sake of illustration. The computer program written is, however, general and all what is required is that the numbering of joints in the same sequencial order.

For K-braced panels, the member data is generated in the similar way. The only difference is the increase in the number of groups in a panel. It is 4 for K-braced instead of 2 for X-braced panels. The typical input data for the panel shown in Fig. 2.4 is given in Table 2.2.

Table 2.2 Typical Data for Generating the Configuration

X-bracing					K-bracing				
J member number	IT*	IP(J)	IQ(J)	A(J)	J	IT	IP(J)	IQ(J)	A(J)
1	1	1	5	...	1	1	1	9	...
5	3	5	2	...	5	2	9	5	...
					13	5	5	2	...
					21	4	5	6	...

* IT = 1 for leg members

= 2 for K-braces

= 3 for X-braces

= 4 for horizontals (4 members)

= 5 for horizontals ~~is~~ in K-braced panel (8 members)

The coordinates of the joints in the basket portion of the tower is fixed and hence generated only once. The connectivity table of the members of the tower remain the same, except that the areas of the member goes on changing. When the area of a member is changed, the latest value will be stored in place of the old value.

Other data read in this subroutine are the width of basket, height of tower body, and height of panels and base width of starting design point.

2.3.2.2 Generation of Variable Data: Subroutine 'XYZCOR'

In the present work, the configuration of the tower body changes during the process of optimization since the panel heights and base width are the design variables. Hence the coordinates of the reference joints changes from design to design. To start with, the coordinates of the reference joints are known which depend upon the starting point of the optimization subroutine. In otherwords the coordinates of the reference joints can be worked out once a set of values for panel heights and base width is known. Knowing the width of basket, width of base and the height of tower body, the slope of leg members in the body can be worked out. With this slope and height of panels, the coordinates of the reference joint can be computed. Once the coordinates of the reference joint is known, the

coordinates of the other joints in that panel can be generated as discussed in the preceding section.

After generating the coordinates of all the joints in the body, the program computes the length of secondary members in each panel. If the length of secondary brace is less than 2.17m, thinnest possible prescribed section is adopted. The area of secondary braces, whose length exceeds 2.17m, is picked up from the list of equal angles so that slenderness ratio of the section taken is less than 250. The weight of all these secondary braces is summed up and is added to the weight of primary members to get the total weight of tower at any design point.

2.4 Static Loads Acting on the Tower

2.4.1 Existing I.S.Codes, IS:802 and IS:875

The transmission line towers are situated mostly in open country. Hence one of the main loads on the transmission line towers is due to natural wind. The natural wind force depends on many factors which include local topography, climate variation, height above ground etc. and as such the wind load should be treated under probabilistic framework. The importance of this necessity can be understood from the recent works on wind loading of structures reported in literature[60-65].

To treat the wind load under probabilistic framework, it is necessary to have statistical data of wind pressures observed around the site under consideration for a considerable period of time. In India, such data is not available, but instead Indian Standard, IS: 875-1964,[66] has recommended for general structural design wind pressure values for different locations of the country. India is divided into three zones for the purposes of presenting the wind map of India. The wind pressure values, including winds of short duration as in squalls, are presented in the form of table for increasing height. The basic wind pressure indicated is the maximum ever likely to occur in the respective areas, under fully exposed conditions.

The Structural Engineering Sectional Committee, constituted in 1967 to draft the code on 'Use of structural steel in overhead transmission line towers: Part I- Loads and permissible stresses'[54], has recommended wind pressure values for the design of transmission line towers based on IS: 875-1964. The wind pressure map and the table giving the pressure values, presented in IS: 802-1967, were taken from IS: 875. Guided by this code on transmission line towers, the towers in India were designed till mid-seventies. This code has prescribed different factors of safety for the design of structural members under normal condition in

which all wires are intact and broken wire condition during which one of the conductors or ground wire is assumed to be broken. The details are presented in the next section.

Later, the sectional committee felt the need to introduce the dynamic effect since wind induce considerable vibration in the tower due to the variation of wind pressure with time. This points to the fact that the wind pressure values to be taken are static equivalent of dynamic wind rather than maximum static pressure. When the code IS: 802 (part I) was revised in 1973 the committee has recommended an increase of about 30 percent in wind pressure values that are presented in IS: 802 (part I)-1967, irrespective of height of the tower. This does not seem to be logical. The stiffness of the tower varies with height and plays an important role to decide the static equivalent of dynamic wind pressure. In other words the increase in static maximum wind pressure, to account for the dynamic effect, has to be arrived at for different heights of tower and cannot be a constant. Hence the design of transmission line tower taking wind pressure values from IS: 802-1973 would result into very conservative design especially for smaller towers. This makes it necessary to analyse the transmission line tower under dynamic wind pressure. Only the dynamic analysis which takes the effect

of stiffness and mass of the tower into account, and wind as a function of time would produce rational design.

Based on the above discussion, loads for the static and dynamic analysis of the tower is derived from IS:802-1967. Comparison of results of Chapters 4 and 5 gives the effect of dynamic wind over the static loading.

The latest code, i.e., IS: 802-1973, refer the intensity of wind pressure in the three zones as light, medium and heavy. The factors of safety recommended remain the same as in the former code, but there are changes in the allowable stresses and allowable maximum slenderness ratios for the members of the tower. Calculation of tower loadings for a typical 132 KV double circuit line tower is also presented in the latest code.

2.4.2 Computation of Loads IS : 802

Transmission line tower is subjected to three types of loads[18,55] viz., (1) vertical loads, (2) transverse loads, and (3) longitudinal loads. The vertical loads are computed on the basis of weight span, shown in Fig. 2.5, which is the distance between the lowest points in the catenary in the spans adjacent to the tower. Transverse loads are due to wind on bare or ice coated wire and also due to change in line deviation. The computation of wind load is based on

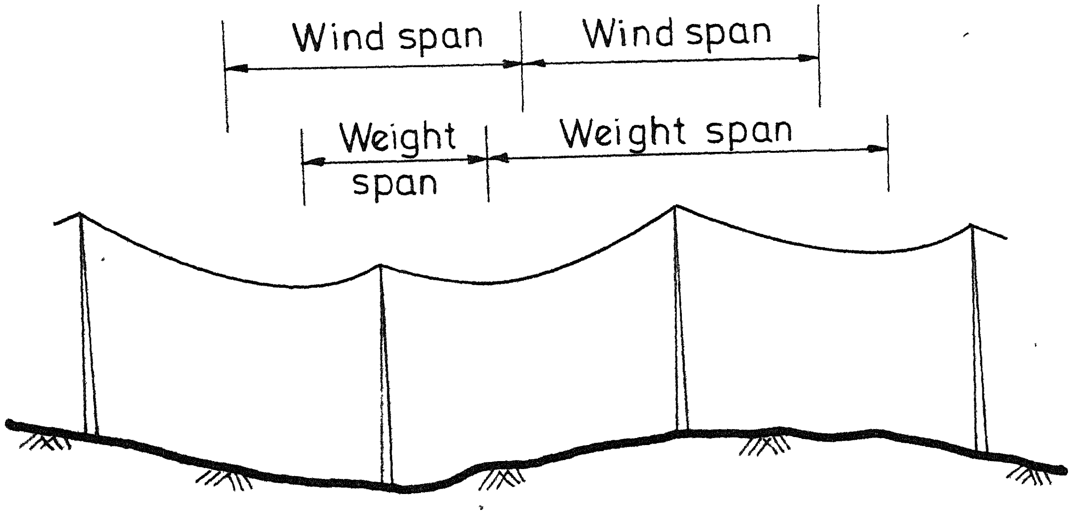


Fig. 2·5 Spans for the computation of wind load and dead load

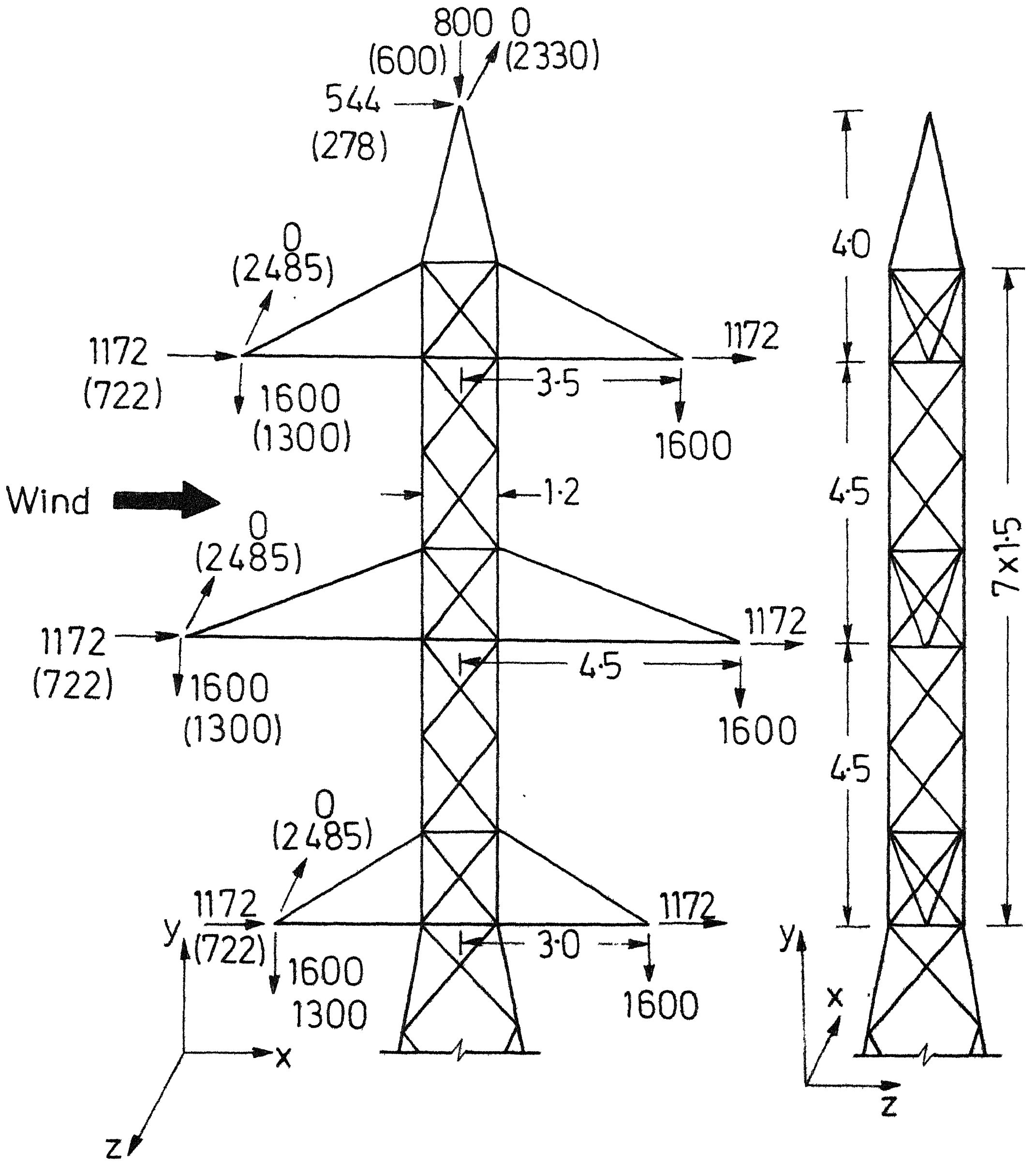
The loading, on the 220 KV double circuit tower (Fig. 1.1), for different cases of normal and broken wire conditions are presented in Fig. 2.6 which are taken for the present work. These loadings are obtained from Cromptons India Ltd. The wind direction is reversible and hence the total number of independent loading conditions turn out to be 8 as follows:

1. Normal condition, i.e., all wires intact and wind blowing.
2. Broken wire condition of ground wire and wind blowing.
3. Broken wire condition of top conductor and wind blowing.
4. Broken wire condition of middle conductor and wind blowing.
5. Borken wire condition of bottom conductor and wind blowing.
6. Same as case 3 but wind in opposite direction.
7. Same as case 4 but wind in opposite direction.
8. Same as case 5 but wind in opposite direction.

However, a simple study reveal that the number of critical loading conditions are much less. This reduces the time required for analysis and hence is a substantial gain for carrying out the optimum design.

2.4.3 Critical Load Conditions

For the study of critical loading conditions, a tower with 15m body height having 63 joints and 208 members,



(Broken wire loads are given in parentheses)
(Loads in kg & dimensions in m)

Fig.2·6 Loads on the tower basket

shown in Fig. 2.7, is analysed subject to all the eight loading conditions as discussed in the preceding section and the self weight as the ninth case. Through this study the following observations are made.

(i) Self weight has considerable effect on lower leg members. This increases the compressive force in the lower leg members on the leeward side and hence has to be considered for analysis to be embedded for the automated optimum design. This is due to the fact that automated optimum design seeking a minimum weight tower endeavours to pick up member areas which will lead the constraints to be bounded.

(ii) Normal condition need not be considered, as this does not produce worst condition of loading for members except the horizontal members at the junction of the cross arms . Furthermore, maximum stress in these horizontal members are at a level for which minimum section prescribed for tower members are safe.

(iii) The broken wire condition of bottom conductor is not critical for any member except those two group of members meeting to form this cross arm. It is, therefore, not necessary to consider this case, for the purposes of optimum design of tower, since the increase in weight due to broken wire condition of this conductor shall be a constant.

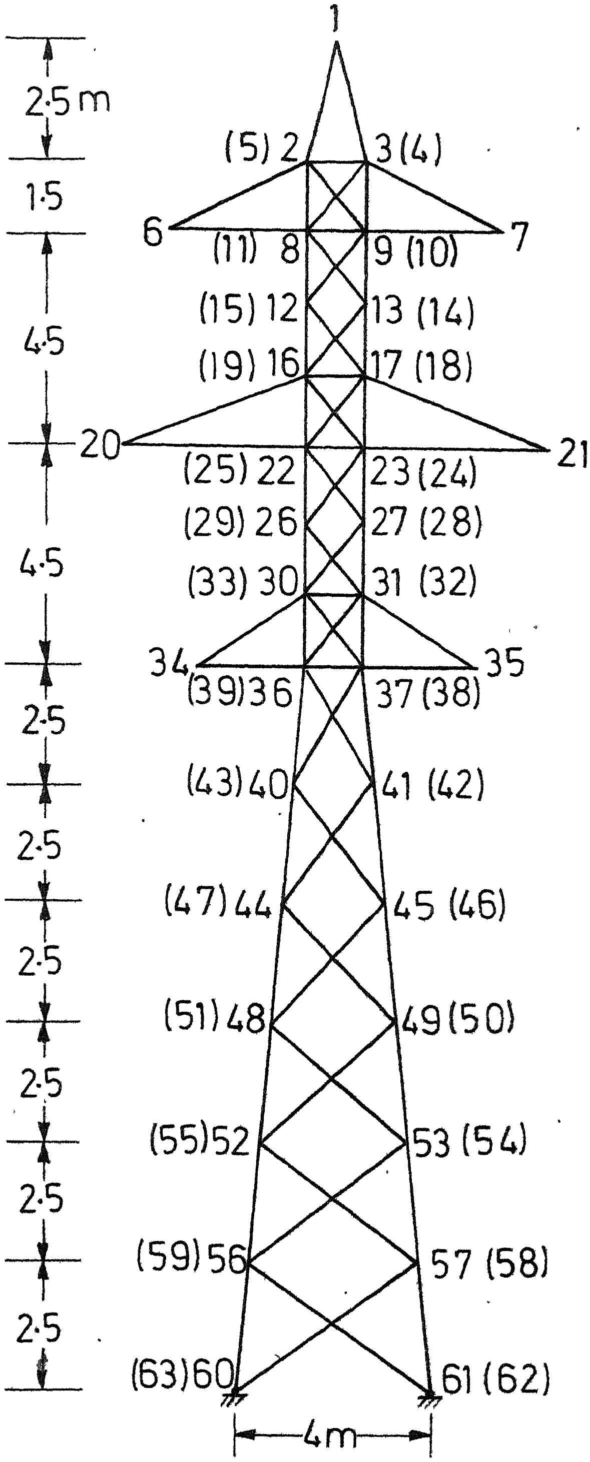


Fig. 2.7 Tower studied under all loading conditions

(iv) Some members (henceforth termed a group in this section) are critically stressed in compression when the wind is blowing from left to right as shown in Fig.2.6 and one of the conductors on the left face snaps. It is observed that when the direction of wind is reversed, then the members in the group experience maximum tensile force whose absolute value is less than the maximum compressive force experienced in the earlier case. Moreover the maximum compressive force in the group in the latter case is less than the maximum compressive force experienced in the former case. Therefore, it is good enough to consider broken wire condition with only wind blowing from it towards the vertical centre line of the tower. Member forces produced due to the eight different loading conditions listed earlier are tabulated for typical members in Table 2.3 in which the values that govern the design are underlined. The member forces listed in Table 2.3 are obtained by multiplying the internal force computed through analysis by the corresponding factor of safety for the loading condition considered. For a few diagonal members the critical compressive force is produced due to loading number 7 or 8. In most of the cases of diagonal braces, this does not govern the design. It is the l/r ratio restrictions, where l is the effective length of the member and r is the least radius of gyration, which govern

Table 2.3 Maximum Member Forces in each Group under Eight Loading Conditions

Joint numbers of a member in the group	Maximum member force (metric ton) due to loading condition*							
	1	2	3	4	5	6	7	8
30, 31	-2.474 2.450	-1.710 3.008	-1.730 2.731	-1.837 2.173	-1.796 2.536	-1.640 2.731	-1.716 2.155	-1.841 2.573
30, 37	-4.858 3.518	-5.684 4.696	-8.186 6.919	-9.643 7.149	-7.406 5.192	-7.587 7.206	-8.120 8.672	-5.594 5.852
36, 37	-2.306 0.928	-2.242 0.653	-2.054 0.665	-1.819 0.701	-3.264 0.695	-2.054 0.634	-1.816 0.660	-3.544 0.679
30, 36	-30.382 21.922	-36.952 30.730	-34.435 24.886	-31.224 18.091	-26.634 18.566	-29.862 27.089	-22.733 23.536	-24.121 19.867
36, 40	-36.292 27.171	-41.878 35.172	-40.448 29.012	-41.309 20.586	-30.089 20.925	-34.589 32.749	-25.860 33.307	-26.668 22.518
36, 41	-2.508 2.052	-3.278 2.945	-7.522 5.068	-9.062 7.253	-7.108 5.972	-5.816 6.913	-8.348 8.513	-6.942 6.503
40, 44	-42.306 31.045	-46.056 37.771	-46.334 31.067	-50.353 25.396	-41.634 26.036	-38.379 37.237	-32.454 41.001	-33.469 32.677
40, 45	-1.257 1.537	-1.804 2.008	-3.105 4.608	-4.443 5.551	-3.659 4.355	-4.235 3.563	-5.215 5.114	-3.984 4.253

* in the same order as listed in Section 2.4.2
\- compression , + tension

the member size.

Based on the above observations it is concluded to analyse the tower only for the three cases of loading listed below.

(1) Ground wire broken condition and wind blowing.

(2) Top conductor broken and wind blowing from it towards the vertical centre line of the tower.

(3) Middle conductor broken and wind blowing from it towards the vertical centre line of the tower.

The basket portion of the tower with the above mentioned critical loading conditions, in addition to the loads corresponding to the normal condition, is shown in Fig.2.6.

2.5 Dynamic Loads Acting on the Tower

Loads on the structure are, in general, dynamic, with an exception of dead loads. Hence the idealization of structural loading, as a function of time is an important step in the present work.

The loads on the transmission line tower can be classified into three groups, viz.,(1) dead loads, (2) broken wire loads, and (3) wind loads. The dead loads, of course, are static loads. The conductor or ground wire breaks during a turbulant wind, say, during heavy cyclonic storm. Hence these broken wire loads are dynamic loads. What

precisely is the load on the tower in this situation is rather complicated[67]. However, IS: 802-1973 has prescribed certain guidelines to treat these broken wire loads as static loads. Hence the broken wire loads are treated like static loads in this work as described in Section 2.4.2. The wind load acting on the tower is a time dependent forcing function and its idealization is the subject of discussion in the following section.

2.5.1 Wind Loads

The wind that passes over the transmission tower is a large scale movement of air, due to thermal currents and the rotation of the earth. The wind velocity which has temporal as well as spatical randomness, depends upon many factors like pressure, temperature, the particles carried by wind etc. In order to design the structure against wind loads a continuous record of temporal and spatial variation of wind, at the site of construction, is essential. But more often than not, this is not possible because of a multitude of reasons. In such a situation, some alternative approach has to be adopted so as to avoid the need of having a continuous wind records.

Maharshi[68] has adopted 'Digital-simulation' technique, in the absence of wind records, to analyse a 175m high T.V. tower. The simulation has been done with Chiu's[69] model as

well as with Shinozouka's[70] model. His work was mainly to study the effect of parameters with different mean wind velocities and compare with that of static wind loading taken at mean value.

To carry out the dynamic analysis with simulated (pseudo-wind) records, the required informations are, (1) mean wind velocity, (2) the standard deviation of wind velocity, and (3) time interval during which the wind is assumed to vary linearly. Even these are not available for most of the locations where failure[71] due to cyclones are estimated heavy. Originally the author was interested to study the response of transmission tower to that cyclonic storm which hit the Indian east coast during November, 1977, resulting in huge loss of life and materials. Quite a number of transmission line towers collapsed in that cyclone. The Indian Meteorological Department, when requested, could provide only the peak values of wind speed observed which are given below. The maximum wind speeds, given below, correspond to the Bay of Bengal storm of 14.11.1977 to 19.11.1977.

(1) Maximum wind speed reported by ship on 17.11.1977 is about 90 knots.

(2) Maximum wind speed estimated from satellite pictures is about 120 knots on 18th and 19th of November, 1977.

(3) Maximum wind speed recorded at Gannavaram (Andra Pradesh) when the storm moved close to the station on 19.11.1977 is 75 knots.

They expressed their inability to provide any wind record or the statistics of wind speed variation. The above informations are as good as that available in IS: 802-1967, since no information regarding the variation of wind speed is available.

With the limitations discussed above, it is decided to carryout the deterministic analysis, in which the wind load is approximated to a known function.

Yeh and Shieh[72] have represented the wind load, for the analysis of hyberbolic cooling towers, by a deterministic load function. The load function assumed has the peak values, at certain time intervals, representing the gust. The load-time history used in their work is shown in Fig. 2.8. The numerical value of peak wind pressure, taken by them, is of no importance here and hence the peak value is represented by p_{max}. Later Steinmetz et al[73] have studied the behaviour of hyberbolic cooling tower for which wind loading was based on full scale measurements by pressure-difference transducers. The pressure readings were taken at the throat of an existing tower located in Martin's creek, Pennsylvania. The dynamic pressure has been represented by

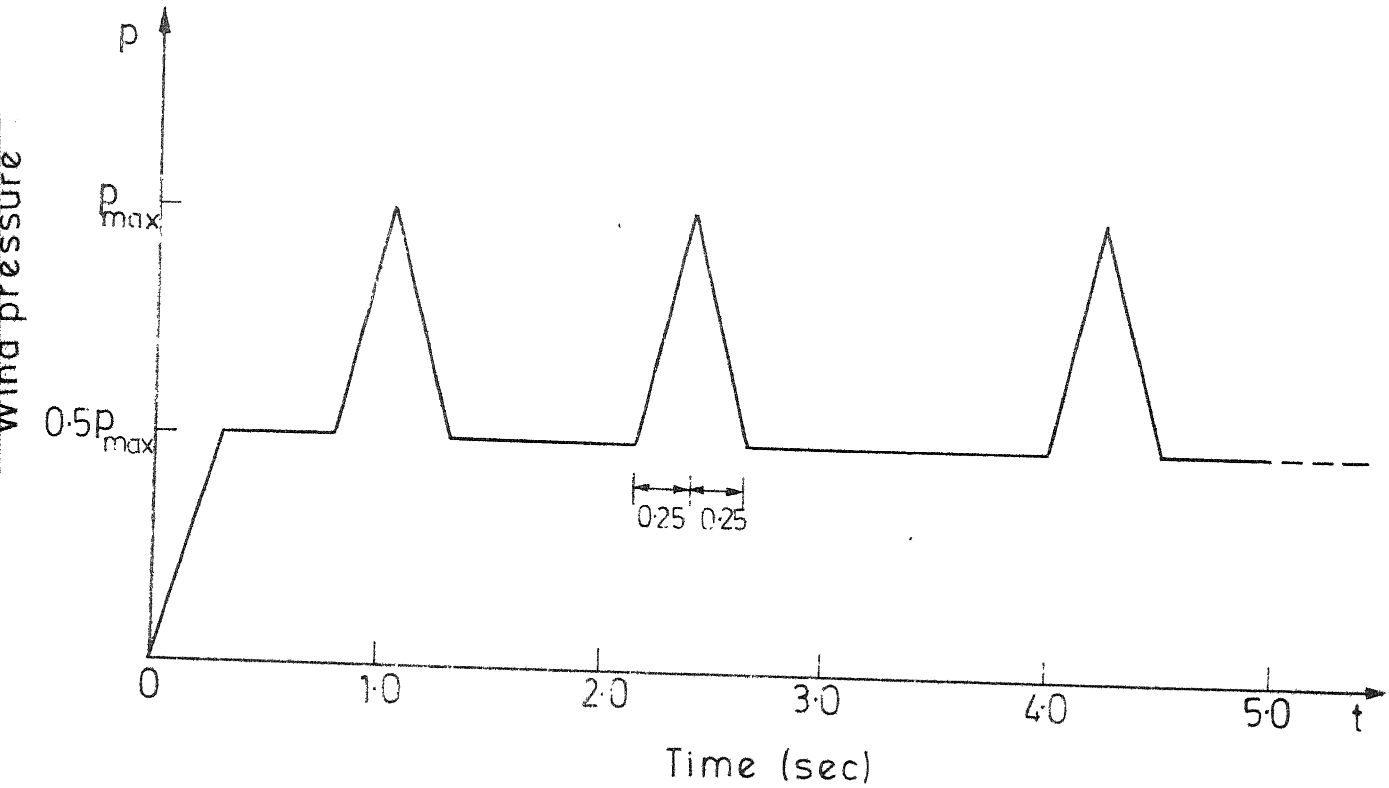


Fig. 2·8 Wind pressure-time history

Fourier series. They found, on comparison, the basic response patterns were similar to Yeh's tower. This shows that load-time history shown in Fig. 2.8 can be used to represent wind without much of error. Hence the deterministic load-time history shown in Fig. 2.8 is adopted for the present study.

Wind pressure varies with height and hence the maximum wind pressure value p_{max} in Fig. 2.8 depends on the height above ground. Based on the wind pressure values in IS: 802-1967, the maximum dynamic wind load, for the transmission tower analysis, has been computed. However, the time history of wind, ~~storm~~ shown in Fig. 2.8, is taken as same for all heights above ground, i.e., time interval of peaks and the level of mean value of wind load remain the same for all dynamic wind loads that are assumed to act at joints of the tower. The wind loads, computed at maximum wind pressure from IS: 802-1967 as well as the other static loads are shown in Fig. 2.6 . In this figure all the loads in the X-direction (wind direction) are dynamic loads for which the time history is shown in Fig. 2.8.

2.5.2 Earthquake Loads

IS: 802 has not mentioned anything about the consideration of earthquake in the design of transmission tower. However IS: 1893-1975[74] has given certain guidelines

for the design of structures in general. The average acceleration spectra given in the above code, is taken for the present work of transmission line tower design. The code has stated the following assumptions applicable for general structural design.

(1) Earthquake causes impulsive ground motion which is complex and irregular in character, changing in period and amplitude and lasting for small duration. Therefore, resonance of the type as visualized under steady state sinusoidal excitations will not occur as it would need time to build up such amplitudes.

(2) Earthquake is not likely to occur simultaneously with wind or maximum flood or maximum sea waves.

The country is classified into five zones for the purpose of determining the seismic forces; the details are available in IS: 1893-1975. For the present study the tower is assumed situated in the most critical zone, i.e., Himalayan region, zone V.

Depending on the structure, the code has recommended two methods for computing the seismic force:

(a) Seismic coefficient method, and

(b) Response spectrum method.

In the present work, the response spectrum method is used, which takes into account the system properties explicitly.

where M is the mass and g is acceleration due to gravity. This horizontal force is assumed to act at the corresponding location of the mass. For the present study the direction of seismic acceleration is assumed in the transverse direction of line. Further details of modal analysis and dynamic stresses in the members of the tower due to earthquake are presented in Chapter 5.

CHAPTER 3

FORMULATION OF OPTIMUM DESIGN PROBLEM AND METHOD OF OPTIMIZATION

3.1 Introduction

When a means for predicting the behaviour of any structure is available, and design philosophy is decided, it is possible to cast the design modification problem in the form of a mathematical programming problem.

A mathematical programming problem is one in which a multivariable function $f(\vec{d})$, where $\vec{d}$ is a n-dimensional vector consisting of d_j, $j=1,2,\ldots n$, is minimized (or maximized) subject to given constraints $g_i(\vec{d}) \{\leq, =, \geq\} b_i$, $i=1,2,\ldots,m$ $(m \leq n)$, where b_i are known constants. There are usually inequality constraints on some or all of d_j. The function $f(\vec{d})$ is called the objective function and its choice is governed by the nature of the problem. For a configuration optimization problem, cross sectional variables of member as well as the variables that define the geometry of the structure are included in vector $\vec{d}$.

The objective of structural design optimization is frequently taken to be the weight minimization. This is justified for transmission line towers which consist of sections made of only one material (generally steel) as well

as the fact that fabrication and erection cost is on tonnage basis. The minimum weight design is, therefore, considered as objective function in the present work.

3.2 Formulation of the Optimization Problem

The general structural optimization problem can be stated as follows:

Minimize $\quad f(\vec{d}) \;,\; \vec{d}^T = [d_1\ d_2 \ldots\ldots d_n]$ (3.1)

Subject to

$$|Y(\vec{d},\vec{x},t)| \leq Y^{(u)} \qquad t \geq 0 \tag{3.2}$$

$$|\sigma(\vec{d},\vec{x},t)| \leq \sigma^{(u)}$$

$$\lambda_i^{(l)} \leq \lambda_i \leq \lambda_i^{(u)} \qquad i = 1,2,\ldots,p \tag{3.3}$$

$$g_k(\underline{\vec{d}}) = c_k \qquad k = 1,2,\ \ldots\ldots,q \tag{3.4}$$

and $\quad d_j^{(l)} \leq d_j \leq d_j^{(u)} \qquad j = 1,2,\ldots.,m$ (3.5)

where f represents the weight of the structure

$\vec{d}$ vector of design variables, viz., geometrical dimensions and cross sectional variables

$|Y(\vec{d},\vec{x},t)|$ represents the displacement at any point on the structure as a function of space variables $\vec{x}$ and time t

$y^{(u)}$ is the upper limit on the displacement

$|\sigma(\vec{d},\vec{x},t)|$ represents the stress at any point on the structure as a function of space variable $\vec{x}$ and time t

$\sigma^{(u)}$ is the upper limit on the stress, taken to be the maximum permissible stress as per design standards

λ_i represents the i-th eigenvalue of the structure

$\lambda_i^{(u)}$ is the upper limit on the i-th eigenvalue

$\lambda_i^{(l)}$ is the lower limit on the i-th eigenvalue

g_i is a function of configuration variables only and hence $\underset{\sim}{\vec{d}}$ is a subset of vector $\vec{d}$

and c_k constant, known from the compatibility requirement.

The behaviour quantities Y, σ, and λ_i are dependent on the design variables and hence the constraint Eqs.(3.2) and (3.3) are called behaviour constraints. The compatibility of structural configuration is imposed by Eq. (3.4). The side constraints, Eq. (3.5), impose the limits on the size of the design variables based on minimum size of sections and any restriction on the geometry of the structure.

For the static loads, the time parameter is set to zero and hence the problem gets simplified as follows:

Minimize $\quad f(\vec{d})\ ,\quad \vec{d}^T = [d_1\ \ d_2\ \ \ldots.\ \ d_n]$

Subject to

$$Y(\vec{d},\vec{x}) \leq Y^{(u)}$$
$$\sigma(\vec{d},\vec{x}) \leq \sigma^{(u)} \tag{3.6}$$

$$g_k(\underline{d}) = c_k \qquad k = 1,2, \ldots, q \tag{3.7}$$

and

$$d_j^{(l)} \leq d_j \leq d_j^{(u)} \qquad j=1,2,\ldots,m \tag{3.8}$$

where various quantities have same physical meaning as described earlier.

3.2.1 Design Variables

As stated in Chapter 2, the basket portion of the transmission tower geometry is defined from electrical considerations. Hence this portion of the tower geometry has little scope for variation with the aim of reduction in weight.

The geometry below the bottom cross arm, ie., body of the tower, is free from any restriction and hence it has ample scope for modification. All that is required is the total height of body should be kept constant through out the process of optimization. The configuration of the tower body is defined from the width of base, number of panels and height of each panel. Hence the design variables chosen in the present work are the panel heights of the tower body and the base width of the tower , Fig. 3.1.

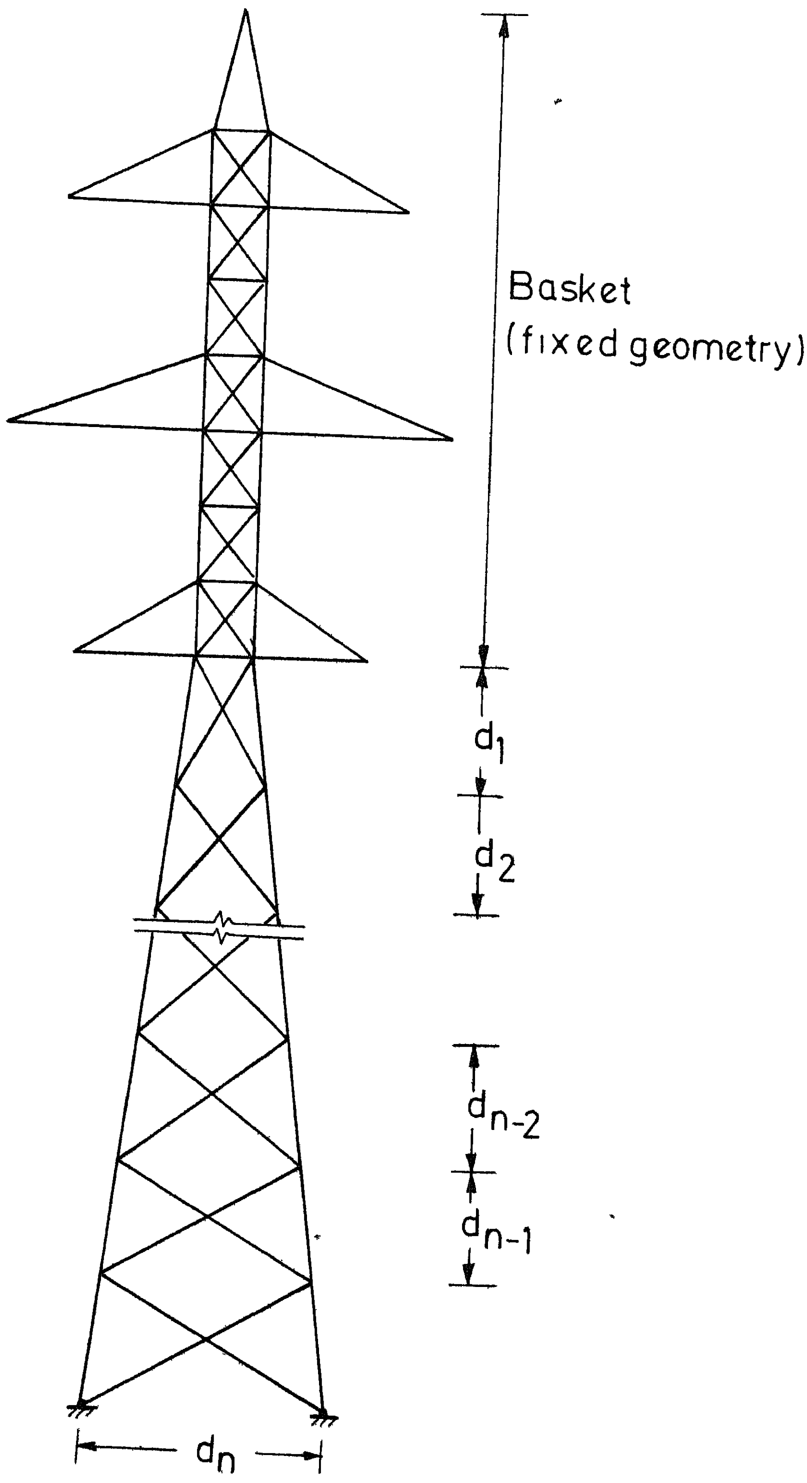


Fig. 3.1 Design variables

Available angle sections are used for the members of the tower. Hence, for the design of members the required section is picked up from the list of available sections and, therefore, the cross sectional variables do not appear explicitly in the optimization process. The details are discussed in Chapter 4.

3.2.2 Objective Function

The goal of the present work is to study the effect of panel heights and the base width on the minimum weight of the tower. Hence the objective function, f, representing the total weight of tower, which is to be minimized, may be stated as follows:

$$f = \rho \sum_{i} A_i \, l_i \, n_i \ , \ i = 1 \text{ to } n_g \tag{3.9}$$

where ρ = density of the material (steel)

l_i = length of members in group i (group defined earlier in page 33)

A_i = area of members in group i

n_i = number of members in group i

and n_g = number of groups which is the sum of groups of primary as well as secondary members.

l_is are length of various members and are implicit function of design variables which are:

(1) the panel height h_i, i=1 to (NP-1), NP being the number of panels in the tower body as shown in Fig. 3.1. Only (NP-1) out of NP panel heights are treated as explicit design variables to make the formulation unconstrained. The details are discussed in the next section,

(2) the base width of the tower, b.

A_is are also implicit functions of the design variables, since they also depend upon the configuration of the tower.

3.2.3 Constraints

The constraints that are to be satisfied, in a general optimization problem, have been listed earlier. However, the following assumptions in the present formulation, under static and dynamic loading conditions, make the optimum design problem unconstrained.

1. The displacement and stresses in the structure are very well correlated, i.e., any constraint on stresses lead to imposition of constraint on displacement implicitly. Further, constraint on displacement is not an important factor in transmission line towers so long as it does not undergo excessive deflection, due to instability of the tower. Therefore, restriction on deflection is **not** necessary. Hence,only constraint on stresses have been considered in the present work. The manner of handling

these constraints, implicitly, is discussed in Section 4.3.

2. In general, the constraint on natural frequency is imposed so that the optimum design obtained has natural frequency away from resonant natural frequencies[75]. However, any design that falls in resonant region will require more weight. Even if the formulation does not impose any restriction on natural frequency, the program will move away from the resonant region, since it is searching for a minimum weight. The results in Chapter 5 brings out this aspect very clearly. Hence in this formulation no restriction on natural frequency is imposed.

3. The body height of the tower is always constant, Eq. (3.4) and Eq. (3.7); i.e., we are attempting the optimum configuration for a given body height of tower. This equality constraint can be stated as,

$$h_1 + h_2 \ldots\ldots + h_{NP-1} + h_{NP} = \text{constant} \qquad (3.10)$$

where h_i is the height of i-th panel from top in body. To handle this constraint implicitly, only the first (NP-1) panel heights are taken as design variables in addition to base width, making the bottom most panel dependent on the value of other panels.

4. Since the available angle sections are used, the section having minimum as well as maximum area stand for

the bounds imposed by Eq. (3.5). The details are discussed in Section 4.3. Hence all the restrictions on the member selection, stipulated by IS: 802-1973, are handled implicitly.

3.2.4 Mathematical Nature of Formulated Problem

The optimization problem formulated in the previous section can be stated as follows:

Minimize $f(\vec{d})$, where $\vec{d}^T = [h_1 \; h_2 \; .... \; h_{NP-1} \; b]$,

since all constraints imposed are being handled implicitly. The unconstrained function, f, which is the weight of the tower to be minimized is not an explicit function of the design vector, $\vec{d}$. However, for a known design vector function value f can be calculated by carrying out the analysis and the design of the tower. Thus the formulation of the optimum configuration design of transmission line tower turns out to be an unconstrained minimization problem.

3.3 Method of Optimization

Several methods are available for solving an unconstrained minimization problem[76]. In the case of simple functions one can discuss the efficiency of a particular method over the other. But for the engineering applications the nature of problem makes the choice limited. The paper by Carpenter and Smith[77] gives a comparative study of unconstrained minimization algorithms, **using** in the Sequencial

Unconstrained Minimization Technique (SUMT)[78], to optimize a variety of structural engineering problems. While listing their conclusions they have rightly pointed out that conclusions concerning computational efficiency of optimization algorithms can never be stated categorically because an exceptional problem can invalidate conclusions drawn from other investigations.

The unconstrained minimization methods can be classified into two broad categories as non-gradient methods and gradient methods. The non-gradient methods require only the evaluation of function values. The gradient methods require, in addition to function values, the evaluation of first and possibly higher order derivatives of the objective function.

The evaluation of gradient could be done using finite difference technique, since the function to be minimized is not an explicit function of design variables. However, this entails additional function evaluations. Apart from the time consumed for such function evaluations, since the area of angle sections that are used for the members of the tower is discrete in nature, the gradient approximation may not be accurate. Hence non-gradient method is the best choice for the problem formulated in the present work.

Among the non-gradient methods, Powell's method[79] is the most widely used search method since it is quadratically convergent. Therefore Powell's method is chosen in the present work for the unconstrained minimization.

3.3.1 Powell's Method

The sequential steps involved in the Powell's method is presented in the form of a flow chart in Fig. 3.2 and discussed briefly below.

(1) Take a starting point $\vec{x}_o$ and linearly independent search directions. To start with, the search directions are taken to be coordinate directions and hence are linearly independent.

(2) Find the step length α^* such that the objective function has minimum value, i.e., $\vec{x}_1 = \vec{x}_o + \alpha^* \vec{s}_n$, by searching along n-th coordinate direction. This design point $\vec{x}_1$ is stored for later use.

(3) For i = 1,2, ... iterate the following steps(4)-(7), till convergence is achieved.

(4) For r = 1,2,....., n, find, along each direction, α_r^* such that $f(\vec{x}_r + \alpha_r^* \vec{s}_r)$ is minimum and update the design vector as $\vec{x}_{r+1} = \vec{x}_r + \alpha_r^* \vec{s}_r$.

(5) Generate the pattern direction given by $\vec{s}_p^{(i)} = \vec{x}_n - \vec{x}_1$. The design point $\vec{x}_n$ is the latest point in step (4).

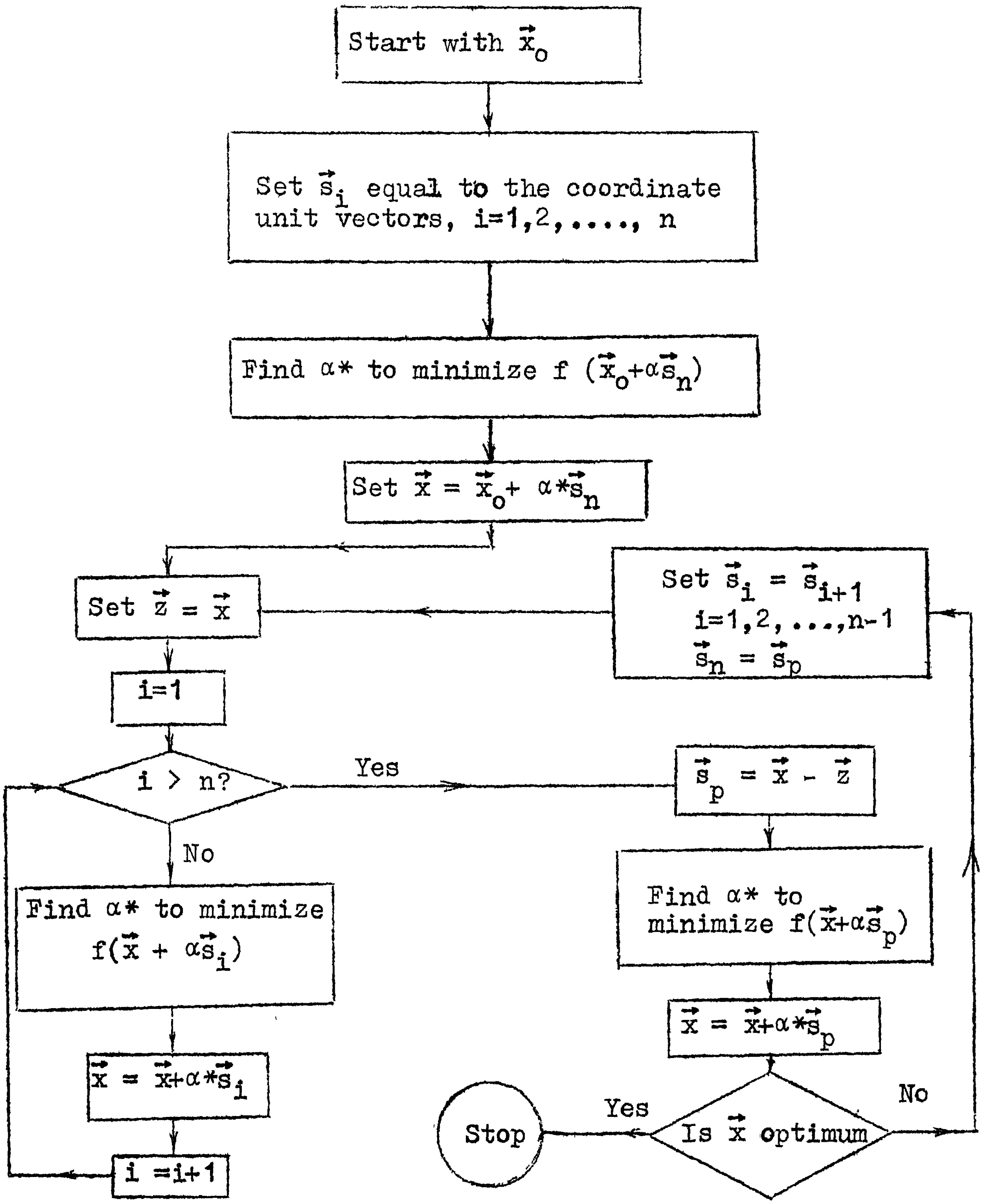


Fig. 3.2 Flow Diagram of Powell's Method

(6) Find α_p^* such that $f(\vec{x}_n + \alpha_p^* \vec{s}_p^{(i)})$ is minimum. Store $\vec{x}_n + \alpha_p^* \vec{s}_p^{(i)}$ as $\vec{x}_1$. Since $\vec{s}_p^{(i)}$ is a better direction than coordinate directions discard one of the coordinate directions. For example in the first cycle $\vec{s}_1$ would be discarded. Renumber the direction vectors, as the first direction is discarded, as follows,

$$\vec{s}_i \leftarrow \vec{s}_{i+1} \text{ and } \vec{s}_n = \vec{s}_p^{(i)}$$

(7) Check for convergence. If optimum is reached, come out of the cycle.

In the above procedure, the routine moves through $\vec{s}_n$ direction twice in the first cycle, steps(2)and(4), and obtains design points $\vec{x}_1$ and $\vec{x}_n$ which are minima along $\vec{s}_n$ and hence the direction $\vec{s}_p^{(1)}$ and similarly subsequent pattern directions have special properties that are the source of rapid convergence of the method. The details are given in the Powell's paper[79] as well as in books on optimization[76,80].

In step(7)of the procedure, the routine checks whether optimum is reached. Powell recommends a termination criterion for ordinary use such that when a cycle produces a change in all variables of less than one-tenth of the required accuracy, the process is stopped. A safer (i.e., less likely to stop prematurely) criterion is also given by

Powell[79] which is much more time consuming.

In the Powell's method, it is necessary to find a optimum step size α^* in each search direction (step 4). For this linear minimization, quadratic interpolation technique[80] is used in the present work as this method does not require any information about the derivative of the function.

In the present work, to find a useable search direction, a step size of $\theta = 0.01$ has been used, to check whether the function decreases along this direction, i.e., $f(\vec{x} + \theta\vec{s}) < f(\vec{x})$. If it is not a useable direction with positive value of θ, θ is taken as -0.01 and the process is repeated. It has been observed, in certain situations, that the function values at $\vec{x} + \theta\vec{s}$ as well as at $\vec{x} - \theta\vec{s}$ are found to be greater than $f(\vec{x})$. However there is difference among the function values evaluated on both sides of the present design point $\vec{x}$. This information is used to establish the search direction, even though it is a violation of basic assumption that the objective function is unimodal. Since the available areas of the angles are discrete in nature, this type of search has been attempted and found useful in few directions.

In the search along a useable direction, objective function values are evaluated at $\alpha = 0.3, 0.6, 0.9, \ldots$ to

bracket the minimum ; i.e., if the direction of search is the coordinate direction, then changes in design variables are 0.3 m , 0.6m,.., etc. Of course, if $f(\vec{x}_r + 0.3\ \vec{s}_r) > f(\vec{x}_r)$ then $\alpha = \frac{0.3}{2}$, $\frac{0.3}{4}$, $\frac{0.3}{8}$, ... etc., is attempted. When α becomes less than 0.02, in the above process, then further search in this direction is terminated and the last value taken as the minimum along this direction.

The optimization process is terminated when the change in all design variables is less than 0.001 in the search along the pattern direction.

The first (n-1) variables of the design vector correspond to panel heights starting from top and the n-th design variable correspond to the base width (Fig. 3.1). Hence to start with , in the first cycle of Powell's method, the routine finds a better base width locally, goes on to panel heights and finally moves along the pattern direction.

CHAPTER 4

OPTIMUM DESIGN FOR STATIC LOADS

4.1 Introduction

Static loads acting on the tower structure have been discussed in Chapter 2. In this chapter, the analysis of tower as an elastic space truss subjected to those static loads is described first. The discrete element method of structural analysis is used in the present work. The discrete elements refer to the primary members of the tower only. The secondary bracing members of the tower are not considered for analysis. However, the effect of secondary braces which subdivide the length of primary members are taken into account for the design of primary members.

The optimum configuration design under static loads is carried out using Powell's method described in the previous chapter. The resulting optimum dimensions for different heights of tower body and the discussion on the results are presented in this chapter.

4.2 Methods of Static Analysis

The analysis of the tower subjected to multiple loading conditions form a significant part of the total

effort required to design the tower. Optimum design process being a recursive analysis-design procedure, one has to perform a large number of analysis of the tower before arriving at the optimum design. Therefore, the choice of an efficient method of analysis is warranted. Hence a study has been made in the present work to choose from among the flexibility and the stiffness methods of structural analysis.

4.2.1 Flexibility Method

A method of analysis useable in design office is given in Ref. 81 which presents a systematic procedure for establishing accurate distribution of shear forces, arising from torsional moments, on to the faces of the space tower. However, this procedure, which falls in the broad category of flexibility method of analysis, is considered not suitable for computerization in the present context. The flexibility method that is programmed for the present purpose of automated analysis is presented below.

The basic procedure for analysing a structural system by the flexibility method is as follows:

i) Release the original structure so that it is a stable determinate structure; the released actions

(reactions and/or internal member actions) are called redundants, q_j, $j = 1,2,\ldots\ n_r$, where n_r is number of redundants.

ii) Calculate the displacements, at the locations of released actions, due to the original loads in the direction of the released actions (redundants). Let these displacements be put in vector form $\vec{d}_{ql}$ where subscripts q and l stand for redundants and applied loads respectively.

iii) Apply unit values of the redundants, one at a time, to the released structure.

iv) Calculate the deformations at each release in the direction of the release due to the unit value of each redundant acting separately.

v) Write a compatibility equation for each released action by describing the displacement of the point of application of the redundants as a linear combination of the deformations calculated in steps (ii) and (iv).

vi) Solve the set of linear algebraic equations for the unknown redundants.

vii) Place the known values of the redundants back on the structure and complete the solution by using the equations of static equilibrium.

The equation of compatibility, in matrix form, along the line of redundants can be written as

$$F \vec{q} + \vec{d}_{ql} = \vec{0} \tag{4.1}$$

where F is the flexibility matrix and $\vec{q}$ is the vector of redundant forces. Eq. (4.1) has n_r symultaneous equations corresponding to n_r redundants. For multiple loading conditions, the vectors $\vec{q}$ and $\vec{d}_{ql}$ become matrices of size $n_r \times n_l$, where n_l is the number of loading conditions. Details of this method can be obtained from standard books on structural analysis[82].

4.2.2 Stiffness Method

The stiffness method is being used extensively to solve structural analysis problems due to the elegant nature of the method from programming point of view. Basically this method consists of writing the equilibrium equations[83] at all the joints in terms of the displacements as

$$K \vec{u} = \vec{p} \tag{4.2}$$

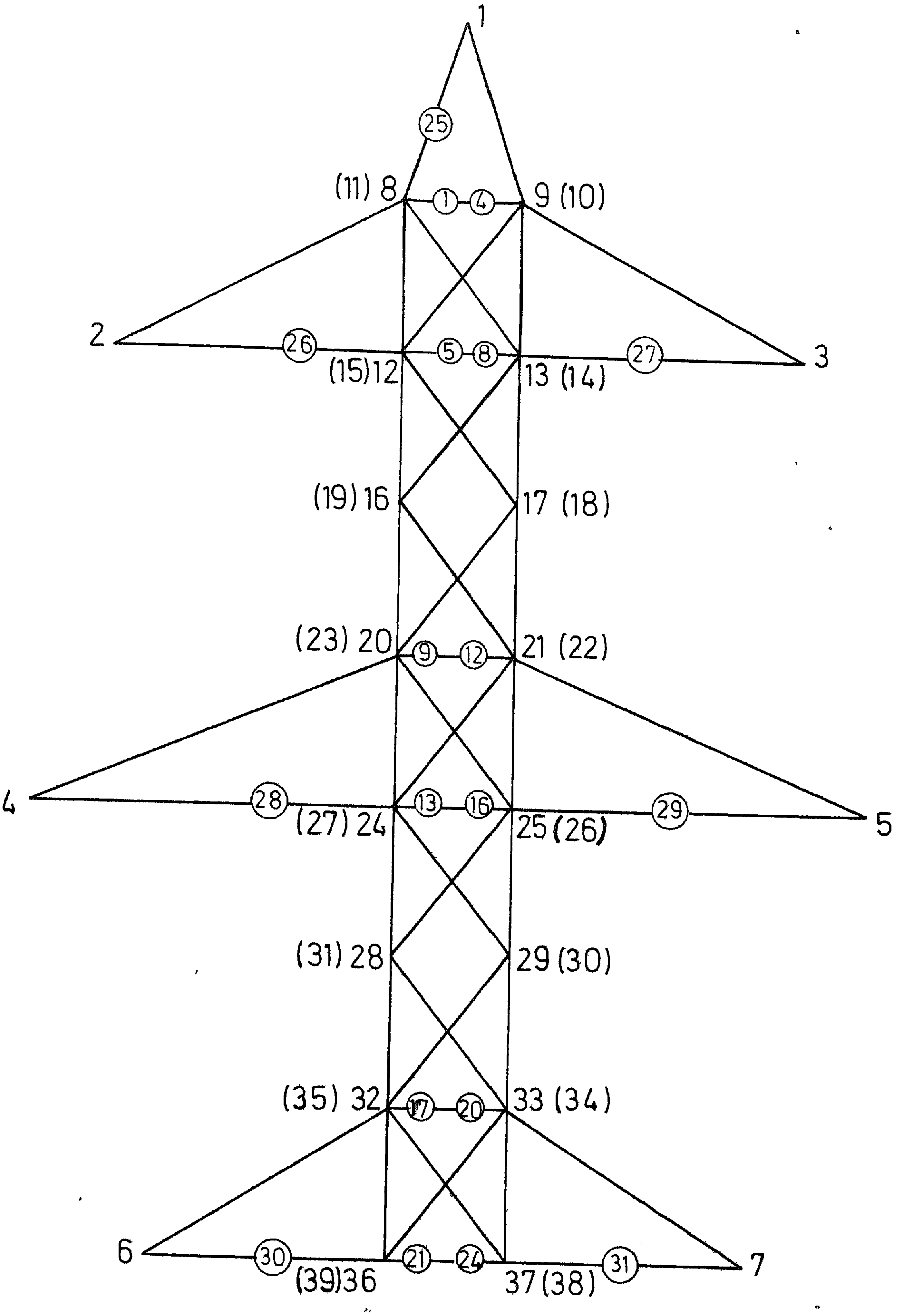
in which K is the assembled stiffness matrix of the structure, $\vec{u}$ is the vector of unknown displacements at the joints and $\vec{p}$ is the known vector of external forces acting on the structure. The displacements can be obtained by inverting the K matrix and post multiplying it by $\vec{p}$. Moreoften, Eq. (4.2) is solved as a set of simultaneous equations[84]

for the unknowns, u_j, $j = 1,2, \ldots n_f$, where n_f is the number of degrees of freedom. Solution as simultaneous equations is less time consuming due to banded-symmetric nature of K matrix. Knowing the vector $\vec{u}$, the internal forces can be obtained from element stiffness matrix[83] and from the components of $\vec{u}$. For multiple loading conditions, vectors $\vec{u}$ and $\vec{p}$ become matrices of size $n_f \times n_l$.

4.2.3 Results Leading to Choice of Method of Static Analysis

The tower shown in Fig. 2.7 has been considered for the study of suitability of flexibility or stiffness method viz-a-viz computer storage and time requirements. It has 31 degrees of indeterminacy. Incidentally all these redundants fall in the basket portion of the tower, Fig. 4.1. The body of the tower has no redundant member when X-type of bracing is adopted.

The program starts from joint 1 and goes on solving the static equilibrium equations at each joint. While resolving the member forces at any joint, in the released structure, the number of unknowns should be only three. Since the equilibrium equations are solved in a computer, the numbering of joints should be done in a particular sequence so as to meet the above requirement. For the present analysis of tower, by flexibility method, the joint numbering in the basket portion is done as shown in Fig. 4.1.



(Redundants are encircled)

Fig.4·1 Redundant members and joint numbering for flexibility analysis

The numbering of joints in the tower body, starting from 40, is as shown in Fig. 2.7.

The number of loading conditions to be considered for the analysis of the tower is 3 as concluded in Chapter 2.

The time taken, on IBM 7044 computer, to analyse this tower for the three loading conditions using flexibility method is 37 seconds.

The same tower, Fig. 2.7, has also been analysed by stiffness method. The number of degrees of freedom, n_f, is 177. The half band width of stiffness matrix is 30 for such double-circuit towers. This method takes only 31 seconds for the analysis, for the three loading conditions. The main core of the computer has been used for storing the stiffness matrix.

It is also observed that stiffness method turns out to be better in computer memory requirements. However, from the point of view of error accumulation, the flexibility method turns out to be superior. This is verified by comparing the resolved internal forces to externally applied loads. The comparison is presented in Table 4.1 for typical joints.

Since the stiffness method is superior both from the view point of computer memory as well as computer time,

Table 4.1 Error Accumulated in Flexibility and Stiffness Methods of Analysis

Joint No. (Fig.2.7)	Flexibility Method			Stiffness Method		
	$\lvert \Sigma F_x \rvert$	$\lvert \Sigma F_y \rvert$	$\lvert \Sigma F_z \rvert$	$\lvert \Sigma F_x \rvert$	$\lvert \Sigma F_y \rvert$	$\lvert \Sigma F_z \rvert$
1	0.00	0.00	0.00	0.01	0.01	0.00
6	0.00	0.00	0.00	0.01	0.02	0.01
12	0.00	0.00	0.00	0.00	0.03	0.01
20	0.00	0.00	0.00	0.02	0.01	0.00
26	0.00	0.00	0.00	0.00	0.03	0.00
34	0.00	0.00	0.00	0.01	0.01	0.00
40	0.00	0.00	0.00	0.00	0.02	0.01
44	0.00	0.00	0.00	0.00	0.01	0.00
48	0.00	0.00	0.00	0.00	0.01	0.00
52	0.00	0.00	0.00	0.00	0.00	0.00
56	0.00	0.00	0.00	0.00	0.01	0.00

Table 4.2 Allowable Maximum Slenderness Ratio

Sl.No.	Type of Member	Limiting L/r Value
1.	Leg members	150
2.	Members carrying computed stresses	200
3.	Redundant members and those carrying nominal stresses	250
4.	Members subjected to axial tension only	500

it is deemed to be more suitable for embedment in the automated optimum design of transmission line tower. Error accumulated in the stiffness method of analysis is negligible for all practical purposes.

4.3 Design of Members

Sectional properties of available angle sections as per Indian Standard Institution[56], commonly used for towers, are stored in the memory of the computer in ascending order of their areas. The first 54 are equal sections and the next 35 are unequal angles. The other sectional properties stored are, (1) least radius of gyration, r_{min}, (2) radius of gyration about x-x axis[56] (parallel to smaller of the two sides) of the unequal angles, r_{xx}, (3) width and thickness of angles, and (4) width of connected side of the unequal angles.

Members of the tower are put into a certain number of groups, as described in configuration generation, Section 2.3.2.1, which is also useful for the design of members. For example all the leg members in a panel form one group and all the diagonal members in the same panel form another group.

After the analysis is carried out, the member forces are compared to get the maximum and minimum forces

among each group of members. These when multiplied by the appropriate factor of safety, presented in Section 2.4.2, gives the design forces of the group. The list of equal angles is then scanned (for diagonal braces, unequal angles starting from 55 in the list) from the beginning. The first section is chosen as the designed section of the group which satisfies the code specifications[55]. They are listed below.

(1) Compressive stress in the member should not exceed the permissible value, given in Appendix 2.

(2) Tensile stress should not exceed 2600 kg/cm^2.

(3) Slenderness ratio of the member adopted not to exceed the values recommended, Table 4.2.

(4) Thickness of angle selected for the members should not be less than 4 mm.

The total weight of all the members, designed to meet the above requirements, is the weight of the primary members. The weight of all secondary braces which has been computed earlier, Section 2.3.2.2, is then added to get the total weight of the tower.

4.4 Results of Optimization

The typical double-circuit tower, Fig. 1.1, subjected to three different critical loading conditions, discussed

earlier (Fig. 2.6), is considered for the optimization study. The following parameters have been kept constant throughout the study.

(1) Plan of basket is 1.2m square and is constant throughout as shown in Fig. 2.6. The height of panels in the basket are not altered.

(2) The base of the tower is square.

(3) The weight of secondary braces in the basket is a constant, since the geometry does not change in this portion of the tower. Hence the weight of these secondary braces is not computed. Thus the weight of the tower indicated on tables and figures does not include the constant weight of secondary braces in the basket portion.

The optimization study is carried out with different starting points to ascertain whether the local minimum is a global minimum. Studies are carried out on 15m, 20m, and 25m tower bodies. The results are presented in Table 4.3 to 4.5 respectively. In these tables the panel heights are in metre and weight in kilogram. The average time taken for one function evaluation (weight of tower) varies from 11.0 to 15.6 seconds on IBM 7044 computer. The CPU time for individual cases are given in the tables.

The reduction in the weight of the tower at the end of each one-dimensional minimization is plotted in Fig. 4.2, for a few representative cases. A close look at the results presented in Table 4.3 to 4.5 and in Fig.4.2 reveal the following:

(1) The variation in the base width of tower has marked effect on the weight of the tower as compared to the variation in panel heights. This observation can be made from the graphs in Fig. 4.2, in which iteration 1 and (n+1) corresponds to base width. The reduction in the weight of the tower in these iterations, i.e., due to change in base width, is appreciable as compared to change in panel heights. This is explainable from the fact that increased base width leads to long members both for the leg as well as for the diagonals. In that case, even though the forces in these members are smaller, the increased length of these members lead to heavier sections.

Moreover, for the diagonal members, it is observed that these are, in general, governed by the slenderness ratio restrictions. Therefore, the compromise between reduced force due to increased base width and reduced length of members due to reduced base width decides the base width corresponding to the minimum weight of the tower. From the optimization study, it is observed that, for the minimum

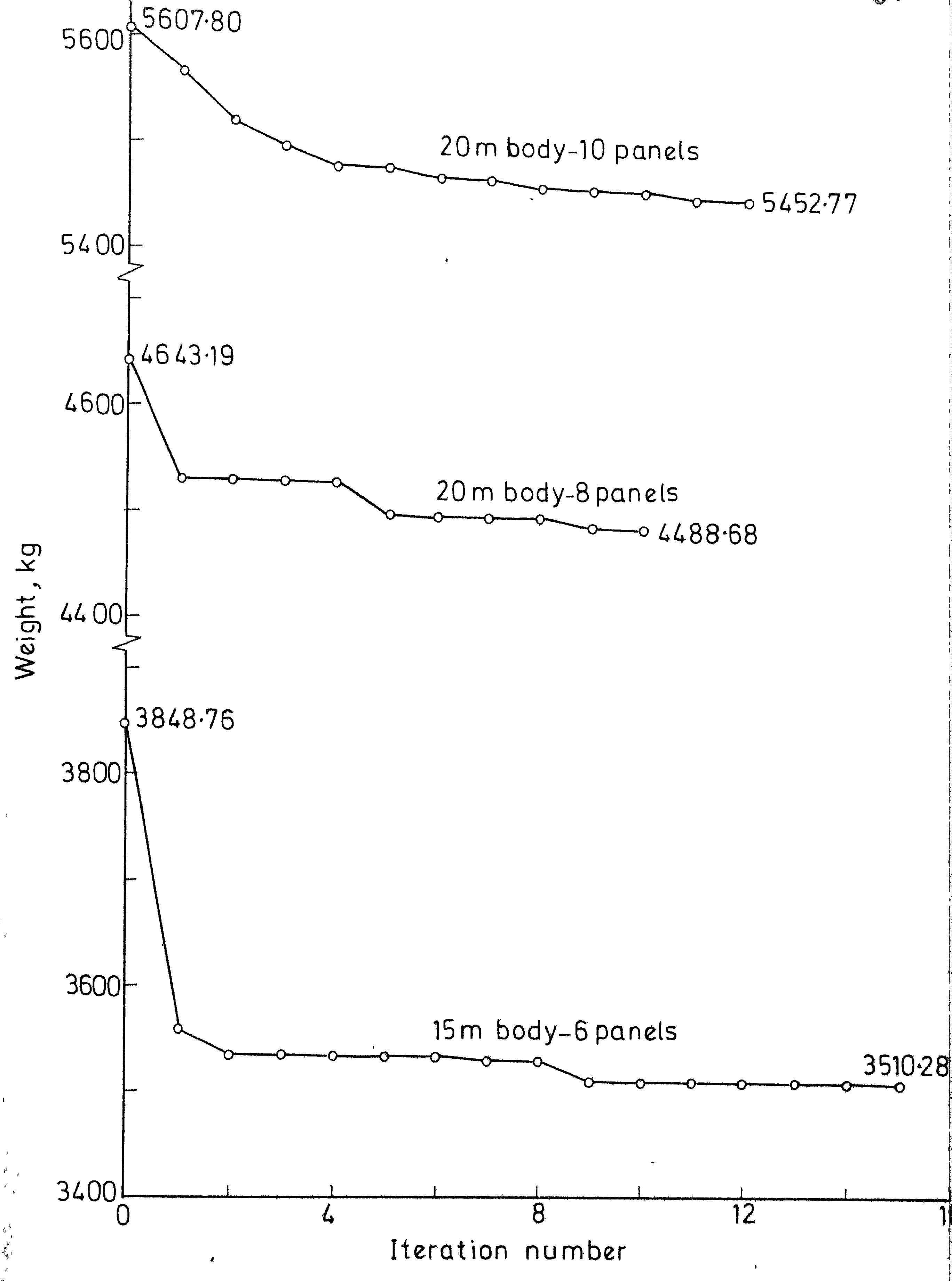
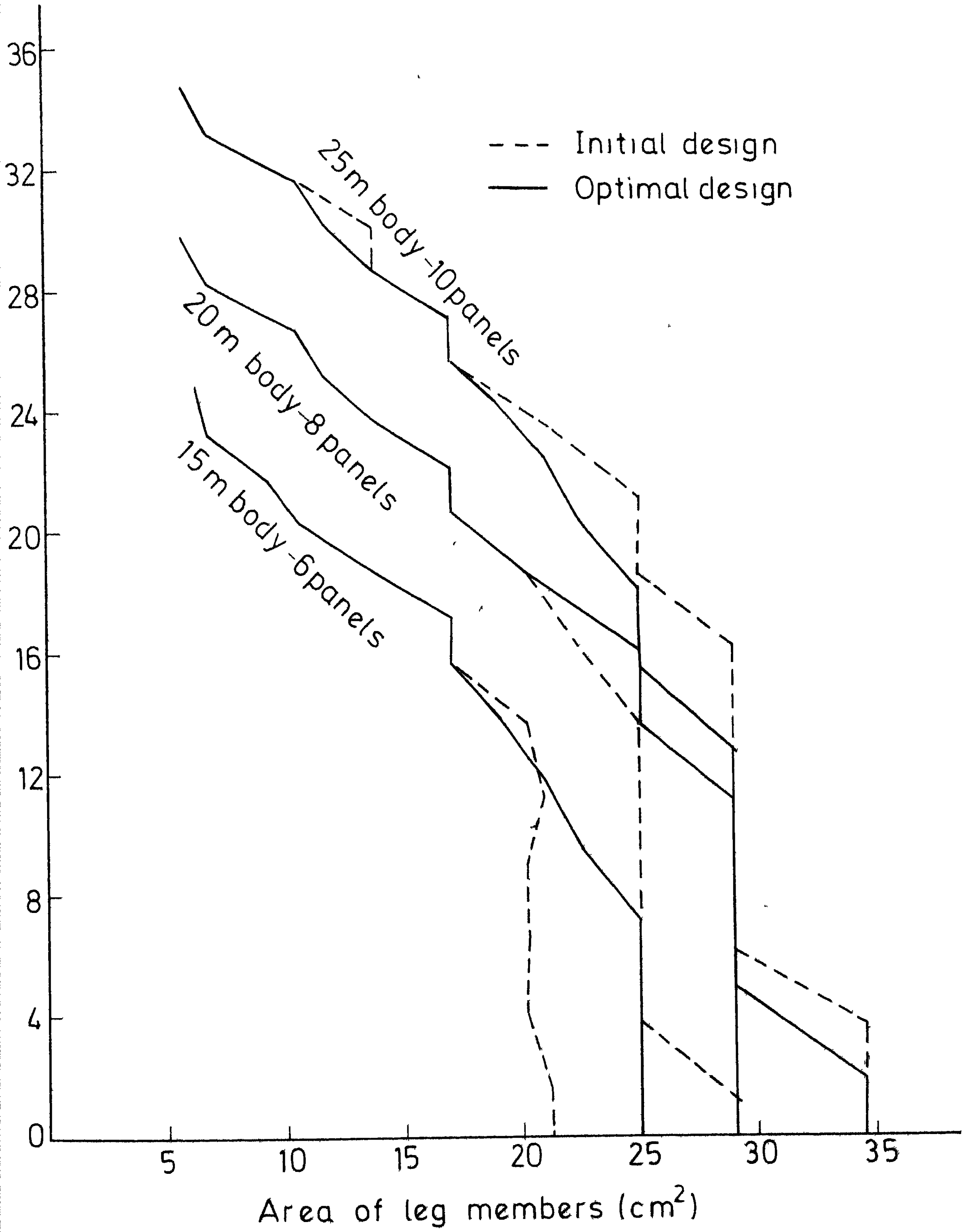


Fig. 4·2 Weight versus iteration (static loading)

weight tower, the base width turns out to be around 4m for body heights of 15m to 25m.

(2) The study further reveals (refer Tables) that for the minimum weight towers, number of panels should be chosen such that the height of the central panel(s) of the body is around 2.5m. For an optimum configuration the panel heights are to be chosen such that the top most is the smallest and the bottom most has the largest panel height. Optimal panel height turns out to be around 2m and 3m respectively for top and bottom most panels. The intermediate panel heights may be varied linearly. This gets confirmed from the results of 25m body tower with 10 panels given in Table 4.5 (columns 6 and 7) where initial design was taken close to the optimum.

(3) To study the changes in stiffness of the tower, the area of leg members at the starting and optimum designs are plotted in Fig. 4.3. The plot correspond to the three towers for which optimum design results are presented in Fig. 4.2. The leg member areas are plotted against the mid-height of panels above ground. It can be observed from this figure that the changes in the member areas in the basket portion are negligible. However, the area of leg members in the body change considerably depending upon the height of tower. For the tower of 15m body height, the



.4·3 Area of leg members along the height of tower (Static loading)

area of leg members in the body increases while the base width decreases (columns 4 and 5, Table 4.3) at the optimum design. This is not so as the height of tower increases. For the tower of 25m body height, the area of leg members decreases while the base width remain almost the same (columns 4 and 5, Table 4.5). This shows that the minimum weight tower is tending to be more flexible as the height of the tower increases. Implicit in this approach of study is the fact that bending stiffness of the tower is primarily dependent upon the stiffness of its leg members.

The above observation warrants the need to check the stability of high tower. It is only this study which shall indicate whether minimum weight obtained is at the cost of overall stability of the tower. Hence the stability of the minimum weight tower with 25m body height has been checked using truss-element geometric stiffness matrix (initial stress matrix), K_G. The total stiffness matrix of the tower is obtained by superimposing K_G and the conventional stiffness matrix, K. The details of the method are described in Chapter 15 of Ref. 83. After elimination of all lower triangular elements of the total stiffness matrix, the diagonal elements are found to be of the order of 10^4. This shows that the determinant $|K+K_G|$

Table 4.3 Results of 15m Body Tower (Static Loading)

	5 panels		6 panels		6 panels		6 panels		7 panels	
	Starting design (S.D)	Optimal design (O.D)	S.D	O.D	S.D	O.D	S.D	O.D	S.D	O.D
Panel heights (m)	3.00	2.50	2.40	2.01	2.50	1.90	2.50	2.42	2.10	1.80
	3.00	2.33	2.40	2.22	2.50	2.17	2.50	2.41	2.10	2.11
	3.00	2.64	2.40	2.37	2.50	2.54	2.50	2.49	2.10	2.06
	3.00	3.18	2.40	2.39	2.50	2.50	2.50	2.50	2.10	2.10
	3.00	3.75	2.40	2.70	2.50	2.08	2.50	2.50	2.10	2.11
			3.00	3.31	2.50	3.81	2.50	2.68	2.10	2.09
									2.40	2.73
Base width(m)	4.00	3.76	5.40	4.21	5.00	4.64	4.00	3.54	5.00	3.49
Total weight (kg)	3712.36	3619.76	3848.76	3510.28	3713.81	3560.76	3630.96	3587.18	3751.75	3505.51
Reduction in weight (percent)	2.49		8.80		4.13		1.20		6.56	
CPU time for one function evaluation on average(sec.)	11.30		11.50		12.15		12.20		12.90	

Table 4.4 Results of 20m Body Tower (Static Loading)

	7 panels		8 panels		9 panels	
	S.D	O.D	S.D	O.D	S.D	O.D
Panel heights(m)	2.83	2.81	2.50	2.47	2.10	1.80
	2.83	2.81	2.50	2.50	2.10	2.06
	2.83	2.82	2.50	2.50	2.10	2.10
	2.83	2.83	2.50	2.73	2.10	2.10
	2.83	2.83	2.50	2.49	2.10	2.10
	2.83	2.83	2.50	2.49	2.10	2.25
	3.02	3.07	2.50	2.50	2.10	2.42
			2.50	2.32	2.10	2.70
					3.20	2.47
Base width(m)	5.40	4.80	4.50	3.64	4.50	4.60
Total weight (kg)	4885.38	4636.80	4643.19	4488.68	4559.81	4499.54
Reduction in weight (percent)	5.10		3.30		1.32	
CPU time for one function evaluation on average (sec.)	12.90		13.75		14.60	

Table 4.5 Results of 25m Body Tower (Static Loading)

	9 panels		10 panels		10 panels	
	S.D	O.D	S.D	O.D	S.D	O.D
Panel-heights (m)	2.80	3.10	2.50	1.30	2.10	2.05
	2.80	2.14	2.50	2.20	2.20	2.18
	2.80	2.78	2.50	2.04	2.30	2.30
	2.80	2.80	2.50	2.50	2.40	2.40
	2.80	2.80	2.50	2.80	2.50	2.50
	2.80	2.80	2.50	2.50	2.50	2.50
	2.80	2.80	2.50	2.84	2.60	2.60
	2.80	2.80	2.50	2.58	2.70	2.70
	2.60	2.98	2.50	2.50	2.80	2.80
			2.50	3.74	2.90	2.97
Base width (m)	4.00	4.05	4.00	4.21	4.20	4.33
Total weight (kg.)	5734.70	5587.88	5607.80	5452.77	5529.37	5452.76
Reduction in weight (percent)	2.56		2.76		1.38	
CPU time for one function evaluation on average (sec.)	14.70		15.60		15.61	

is very high (>> 1) and hence the tower is quite stable. Therefore, it is not necessary to embed stability consideration in the framework of optimum design of these towers. However a final check on the overall stability of the minimum weight tower is still recommended.

(4) It is also observed, from the function values (weight of tower), that the reduction in weight after first cycle of minimization is negligible. The graph for 15m body in Fig. 4.2 gives the function values through two cycles. A similar tendency is noted for higher towers also. So it is good enough to stop the optimization process after moving through the first pattern direction.

(5) The results presented in Table 4.3 to 4.5 gives an idea of convergence to local minima. All the three designs, tabulated for 15m body tower with 6 panels, have converged to the minimum weight of around 3550 kg from three different starting points. Similar trend is noticed for 20m and 25m tower bodies. From these it can be concluded with some degree of confidence that the minimum weight reported herein corresponds to the global minimum.

4.5 Recommended Rational Sections

From the angles picked up for the members of the tower, for all the results presented in the previous section,

it is observed that certain angles are always avoided from selection. This raised the suspicion that some of the available angles are not rational. A look into the table of sectional properties reveal one of the possible reasons: when the angles are listed in the ascending order of their areas (Table A2.1), the other sectional properties, especially the least moment of inertia, do not follow the same distribution as that of the area. Based on this observation a study has been conducted. The study resulted in the recommendation of rational angles for the members of any truss structure. The detailed discussion and the list of recommended angle sections are presented in Appendix 2.

For the recommended rational angles, the area as well as the least moment of inertia have stepwise increase in their values. Using these recommended angles, in place of the available I.S. angle sections, the optimum design of transmission line towers is carried out. The starting configuration is chosen based on the results obtained using available angles, in the previous section. The results of this study are presented in Table 4.6 for 15m, 20m, and 25m body of the tower. A further reduction of 2.75 percent is achieved for 20m and 25m body heights by using suggested rational sections. However the reduction in weight for 15m tower body is negligible, i.e., minimum weights obtained

using I.S. angles as well as the recommended angles are nearly the same. This is due to the fact that for small area range, the rational angles recommended are also available in ISI Handbook (refer to areas in Table A2.1 and A2.2), which are required for 15m tower body. This is not so for large area range and hence the reduction in weight of the tower is considerable for 20m and 25m body heights.

Results of Table 4.6 further reveal that the optimum base widths of tower obtained are less than those obtained with available angles. The optimum base widths obtained vary from 3.60m to 3.90m for the range of heights considered in the present work. There is no change in the conclusions arrived already in respect of panel heights.

CHAPTER 5

OPTIMUM DESIGN FOR DYNAMIC LOADS

5.1 Introduction

Structural design for dynamic loads consists of three steps, viz., (1) idealization of structural loading as a function of time, (2) study of structural characteristics via modal analysis, and (3) response prediction inorder to evolve a safe design. The first step of load idealization is already described in Chapter 2. The present chapter starts with modal analysis of transmission line tower followed by the description of response prediction through modal superposition method. Optimum configuration design under dynamic loads is then carried out. The results of dynamic analysis and optimum design are tabulated and discussed in this chapter.

5.2 Modal Analysis

The equation of motion for a system of structural elements under dynamic loading can be written as

$$M \ddot{\vec{u}} + C \dot{\vec{u}} + K \vec{u} = \vec{p} \tag{5.1}$$

where M, C and K are respectively mass, damping and stiffness matrices of the system; $\vec{u}$, $\dot{\vec{u}}$, $\ddot{\vec{u}}$ and $\vec{p}$ are the displacement velocity, acceleration and load vectors respectively. In

structural application, the K and M matrices are the result of the finite element idealization[85] of the actual structure. The size of matrices involved is $n_f \times n_f$ where n_f is the number of degrees of freedom of the system.

The equation of motion for the free vibration of the structure is obtained from Eq. (5.1) by setting the right hand side as zero vector and assuming the system undamped as

$$M \ddot{\vec{u}} + K \vec{u} = \vec{0} \tag{5.2}$$

Substituting for the displacements,

$$\vec{u} = \vec{\phi} \sin\omega (t - t_0), \tag{5.3}$$

we obtain the generalized eigenvalue problem

$$K \vec{\phi} = \omega^2 M \vec{\phi} \tag{5.4}$$

The n_f eigenvalues give the natural frequencies of the system and the eigenvectors are the corresponding modes of vibration. The complete solution to Eq. (5.4) can be written as

$$K \Phi = \Omega^2 M \Phi \tag{5.5}$$

in which the columns in Φ are the eigenvectors $\vec{\phi}_i$ and $\Omega^2 = \text{diag.}(\omega_i^2)$.

The basis is now changed from the physical coordinate system which was used to establish Eq. (5.1) to

the M-orthonormal basis of eigenvectors. Using

$$\vec{u} = \Phi \vec{q} \tag{5.6}$$

where the vector, $\vec{q}$, list the coordinates in the new basis. The equilibrium equations, Eq. (5.1), become

$$\ddot{\vec{q}} + \Delta \dot{\vec{q}} + \Omega^2 \vec{q} = \Phi^T \vec{p} \tag{5.7}$$

in which $\Delta = \Phi^T C \Phi$ and is assumed to be diagonal. This requires that the damping matrix is of a restricted form[86]. Eq. (5.7) consists of n_f uncoupled equations which can readily be solved[87-89].

5.2.1 Eigenvalue Problem

The most time consuming step in the dynamic analysis is the solution of the eigenvalue problem, Eq. (5.5). If the order of the matrices is large, the computer time required to solve for all the eigenvalues and vectors can be enormous. However, the structure may respond primarily in only a few modes from the lowest end of the spectrum and the contribution of the other modes may be negligible. In this case a method of solution is needed which calculates only the required frequencies and mode shapes economically.

The detailed theory on matrix eigenvalue problem is available in Ref. 90. However, in structural applications the approach is slightly different. Based on the structural

stiffness properties, minimum energy principle, and from the experience gained to date, two approaches emerge for partial eigensolution of large systems, viz.,

(1) Polynomial iteration method

(2) Subspace iteration method.

In the first method, the zeros of the characteristic polynomial, $p(\lambda)$, given by

$$p(\lambda) = |K - \lambda M| \tag{5.8}$$

is searched in an iterative manner. Hence this method is advantageous only for systems having small band width[91-93]. On the other hand, the subspace iteration method, can be used effectively for large banded systems[94,95]. This method uses the Rayleigh minimum principle. Most of the methods for partial eigensolution can be shown to fall in the latter category.

For the size of matrices involved in the present study, discussed in Chapter 2, subspace iteration method is considered most suitable. The details are described in the following section.

5.2.2 Method of Solution of Eigenvalue Problem: Subspace Iteration Method

The subspace iteration method[96], in essence, consists of the following three steps:

(1) Establish q starting iteration vectors, $q \geq p$, where p is the number of eigenvalues and vectors to be obtained.

(2) Use simultaneous inverse iteration on the q vectors and Ritz analysis to extract the 'best' eigenvalue and eigenvector approximations from the q iteration vectors.

(3) After convergence to the subspace, use the Sturm sequence check to verify that the required eigenvalues and corresponding eigenvectors have been extracted.

The solution procedure has been called subspace iteration method, because the iteration is equivalent to iterating with a q-dimensional subspace and should not be regarded as a simultaneous iteration with q individual vectors.

The basic objective in the subspace iteration method is to solve for the lowest p eigenvalues and the corresponding eigenvectors satisfying

$$K \Phi = \Lambda M \Phi \qquad (5.9)$$

where $\Lambda = \mathrm{diag}(\lambda_i)$ and $\Phi = [\vec{\phi}_1, \ldots, \vec{\phi}_p]$.

The essential idea of the subspace iteration method uses the fact that the eigenvectors in Eq. (5.9) form an M-orthonormal basis of the p-dimensional least-dominant subspace of the operators K and M, which is called

E_∞. In the solution, the iteration with p linearly independent vectors can therefore be regarded as an iteration with a subspace. The starting iteration vectors span E_1 and iteration continues until the converged solution theoretically spans E_∞. The fact that iteration is performed with a subspace has some important consequences. The total number of required iterations depends on how 'close' E_1 is to E_∞, and not on how close each iteration vector is to an eigenvector. Hence the effectiveness of the algorithm lies in that, it is much easier to establish a p-dimensional starting subspace which is close to E_∞ than to find p vectors where each is close to a corresponding eigenvector. Also, because iteration is performed with a subspace, convergence of the subspace is all that is required and not of individual iteration vectors to eigenvectors. In other words, if the iteration vectors are linear combinations of the required eigenvectors, the solution algorithm converges in one step.

The algorithm presented below is the procedure of step(2)of subspace iteration method.

For $k = 1,2,....,$ iterate from E_k to E_{k+1} as follows:

(1) Carry out the inverse iteration,

$$K \bar{X}_{k+1} = M X_k \qquad (5.10)$$

where X_1 is the matrix consisting of the set of assumed starting iteration vectors and $\bar{X}_{k+1}$ is the

intermediate improved set of vectors for Ritz analysis. The constraint on the choice of X_1 is that none of its vectors be orthogonal to the corresponding eigenvectors.

(2) Find the projections of the operators K and M onto E_{k+1}:

$$K_{k+1} = \bar{X}_{k+1} \; K \; \bar{X}_{k+1} \qquad (5.11)$$

$$M_{k+1} = \bar{X}_{k+1} \; M \; \bar{X}_{k+1} \qquad (5.12)$$

(3) Find the eigensolution of the projected operations, i.e., solve

$$K_{k+1} \; Q_{k+1} = \Lambda_{k+1} \; M_{k+1} \; Q_{k+1} \qquad (5.13)$$

(4) Find an improved approximation to the eigenvectors as

$$X_{k+1} = \bar{X}_{k+1} \; Q_{k+1} \qquad (5.14)$$

Now

$$\left.\begin{array}{ll} & \Lambda_{k+1} \longrightarrow \Lambda \\ \text{and} & X_{k+1} \longrightarrow \Phi \end{array}\right\} \text{as } k \longrightarrow \infty$$

In the subspace iteration, it is implied that the iteration vectors are ordered in an appropriate way, i.e., the iteration vectors $\vec{\phi}_1$, $\vec{\phi}_2$, ..., are stored as the first, second,...., columns of X_{k+1}.

The effectiveness of the algorithm lies to a large extent in that a few iterations can already give good approximations to the required eigenpair. The reason is

that one subspace iteration in Eq. (5.10) to Eq. (5.14) is, in fact, a Ritz analysis described in Appendix 3. In other words, the eigenvalues from lowest end of the spectrum are approximated best in the iteration and all eigenvalue approximations are upper bounds[97,98] on the actual eigenvalues sought.

A notable observation in the subspace iteration method described above is that K_{k+1} and M_{k+1} in Eq. (5.11) and Eq. (5.12) respectively, tend towards the diagonal form as the number of iterations increases; i.e., K_{k+1} and M_{k+1} are diagonal matrices when the columns in X_{k+1} store multiples of eigenvectors. Hence the generalized Jacobi method, presented in Appendix 4, is used very effectively for the solution of the eigenproblem in Eq. (5.13). The size of matrices in Eq. (5.13) is only $p \times p$.

An important aspect is the convergence of the method. Assuming that in the iteration the vectors in X_{k+1} are ordered in such a way that the i-th diagonal element in Λ_{k+1} is larger than the (i-1)th element, i=2, ...p, the i-th column in X_{k+1} converges linearly to $\vec{\phi}_i$ and the convergence rate is λ_i/λ_{p+1}. Although this is an asymptotic convergence rate, it indicates that the smallest eigenvalues converge fastest. In addition, a higher convergence rate can be obtained by using q iteration vectors, with $q > p$. However, using more

iteration vectors will only increase the computer effort for one iteration.

It is apparent that the closeness of E_k to E_∞ is the measure of convergence which can be quantified by the computed eigenvalues (or eigenvectors). Let the computed eigenvalues in the iterations (k-1) and k respectively be $\lambda_i^{(k-1)}$ and $\lambda_i^{(k)}$, then the ratio

$$\varepsilon = |\lambda_i^{(k)} - \lambda_i^{(k-1)}| \,/\, \lambda_i^{(k)} \tag{5.15}$$

is used as a measure of convergence. For example, for the eigenvalues to be accurate to about 4 digits, it is necessary to iterate until this ε is less than 10^{-5}.

The resulting eigenvalues are checked by using the Sturm sequence property[90]. This property states that in Gauss elimination to evaluate $LDL^T = K - \mu M$, in which μ is the shift and L is the lower triangular matrix, the number of negative elements in the diagonal matrix D is the number of eigenvalues smaller than μ. In order to use a meaningful μ, it is necessary to find bounds for the exact eigenvalues λ_i, using the calculated values $\lambda_i^{(k)}$. A conservative estimate for a region in which the exact eigenvalues lie is given[94] by

$$0.99\lambda_i^{(k)} < \lambda_i < 1.01\,\lambda_i^{(k)} \tag{5.16}$$

Further details of the method and the computational time

estimates are available in Ref. 96.

5.2.3 Eigensolution of Transmission Line Tower

5.2.3.1 Starting Subspace

The first step in the subspace iteration method is the selection of the starting iteration vectors, X_1. The starting vectors for the present problem of transmission line tower is obtained from static deflection shapes. However, for the general structural application, the selection of starting vectors can be chosen based on the elements of K and M matrices[96].

The transmission line tower has been modelled as n_f degrees of freedom system ($n_f = 3n$ where n is number of joints, above the fixed base of the tower). Thus a complete eigensolution will consists of n_f eigenvalues and associated mode shapes. However, a reasonable accurate dynamic analysis of the system can be performed by estimating only first few eigenvalues and associated eigenvectors from the lowest end of the spectrum. For this purpose the tower can be thought of as a tapered cantilever beam in space having almost equal stiffness in the transverse and longitudinal direction. The actual variation in stiffness along these two direction is due to cross arms. Such a tapered cantilever beam is capable of having principal modes of vibration along the transverse direction, longitudinal direction and about

centroidal axis. The principal modes of vibration along transverse and longitudinal directions are bending modes while those about the centroidal axis the torsional modes. The first two principal modes along the transverse direction as well as about the centroidal axis for a cantilever beam are shown in Fig. 5.1. The corresponding static deflection curves for the transmission line towers are obtained by applying suitable static loads as follows:

The first mode of Fig. 5.1(b) is approximated by applying a concentrated load at the tip of the tower along the transverse direction. The first mode in the longitudinal direction is approximated, in a similar way, by applying a concentrated load at the tip of the tower in the longitudinal direction. The second mode of Fig. 5.1(b) is once again approximated by applying two concentrated loads one at the tip of the tower and the other 3 times in magnitude[99] of that at the tip, applied at the level of bottom cross arm of the tower, in the opposite direction. The first mode of Fig. 5.1(c) is approximated by applying a couple at the top conductor points of the tower. The second mode of Fig. 5.1(c) is approximated by applying two couples, of same magnitude, one at the top conductor points and the other at the bottom conductor points in the opposite direction. All these static loads are shown in Fig. 5.2.

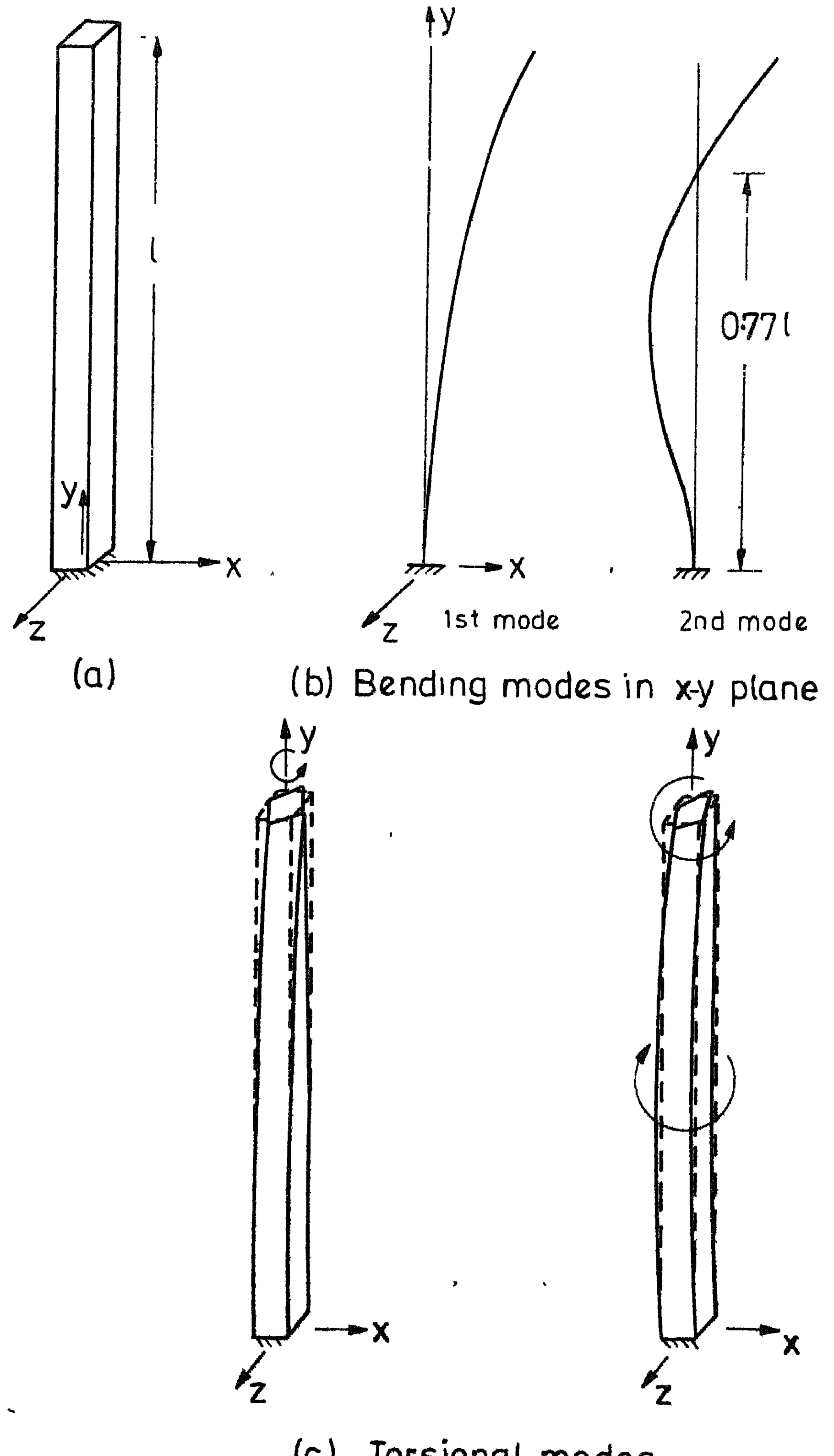


Fig.5·1 Typical mode shapes of a cantilever beam

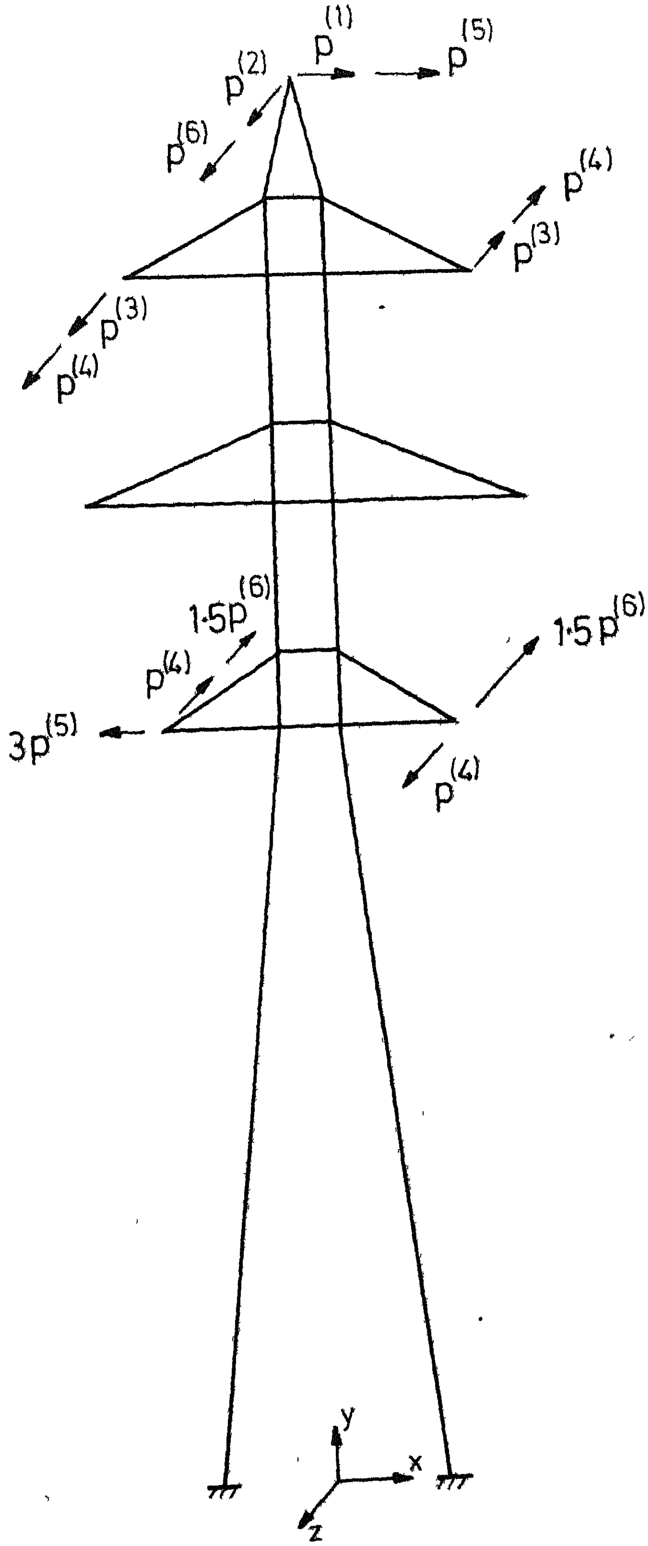


Fig.5·2 Loading conditions

[Deflected shape corresponding to load condition $p^{(j)}$ as starting vector to extract j-th mode shape]

The starting iteration vectors, obtained from static deflection shapes, are to be arranged in such a way to get the eigenpair convergence in the increasing order of their eigenvalues. This can be easily achieved by trial and error method or by solving a series of individual problems and arranging the eigenvalue solutions in ascending sequence.

Six eigenmodes in the increasing order of eigenvalues as listed below have been extracted in the present work.

1. First bending mode in the transverse direction.
2. First bending mode in the longitudinal direction
3. First torsional mode
4. Second torsional mode
5. Second bending mode in the transverse direction
6. Second bending mode in the longitudinal direction.

5.2.3.2 Results

A tower of 15m body height is chosen for the study of eigenpair. The base width of tower is 5.4m. Number of panels in the body has been taken as 6. The size of the stiffness matrix is 177 square and its half band width is 30. Lumped mass formulation is used and hence mass matrix turns out to be diagonal.

The masses lumped at the joints are very small and this posed computational problem. To overcome this problem

scaling has been applied for the elements of the mass matrix as described below:

Consider the eigenvalue problem

$$K \vec{\phi} = \lambda M \vec{\phi} \qquad (5.17)$$

Rewriting the above equation, with the introduction of a constant c, we get,

$$K \vec{\phi} = \frac{\lambda}{c} (c\ M) \vec{\phi}$$

i.e.,

$$K \vec{\phi} = \bar{\lambda} \bar{M} \vec{\phi} \qquad (5.18)$$

The eigensolution of Eq. (5.18) can be obtained without any computational difficulty, since the elements of $\bar{M}$ are c M , c being any suitable constant. The eigenvectors, that are obtained from the solution of Eq. (5.18) , do not change where as the eigenvalues so obtained have to be multiplied by c to get the eigenvalues of Eq. (5.17).

To get a suitable value for the constant c, the eigenproblem of the transmission tower, under discussion, has been studied with different values for c. The results of this study are presented in Table 5.1(a) , (b) and (c).

From these tables, it is clear that the scaling constant of 1000 and 10000 have pruduced equally good convergence, whereas it is slow with c = 100 especially for higher eigenvalues. So the scaling constant of 1000 has been used for the eigenpair extraction.

Table 5.1(a) Results of Eigen-Iteration with Scaling Factor c = 100

Iteration No.	Approximation to Eigenvalues of the given problem, Eq.(5.17)					
	1	2	3	4	5	6
1	844.68	874.08	1619.36	7319.56	7962.39	9063.24
2	844.66	874.06	1617.43	7125.40	7957.30	9051.26
3	844.65	874.06	1617.28	7104.52	7957.20	9051.02
4	844.66	874.06	1617.42	7104.19	7957.20	9051.13
5	844.66	874.06	1617.41	7103.90	7957.17	9051.13
6	844.66	874.06	1617.33	7103.84	7957.96	9051.13
15	844.66	874.06	1617.42	7103.88	7957.10	9051.19
16	844.66	874.06	1617.41	7103.79	7957.07	9051.23

Table 5.1(b) Results of Eigen-Iteration with Scaling Factor c=1000

Iteration No.	Approximation to Eigenvalues of the given problem, Eq. (5.17)					
	1	2	3	4	5	6
1	844.69	874.07	1619.36	7319.56	7962.44	9063.14
2	844.66	874.06	1617.43	7125.46	7957.30	9051.26
3	844.66	874.06	1617.42	7106.34	7957.21	9051.13
4	844.66	874.06	1617.42	7104.19	7957.21	9051.13
5	844.66	874.06	1617.42	7103.90	7957.21	9051.13
6	844.66	874.06	1617.42	7103.86	7957.21	9051.13

Table 5.1(c) Results of Eigen-Iteration with Scaling Factor $c = 10000$

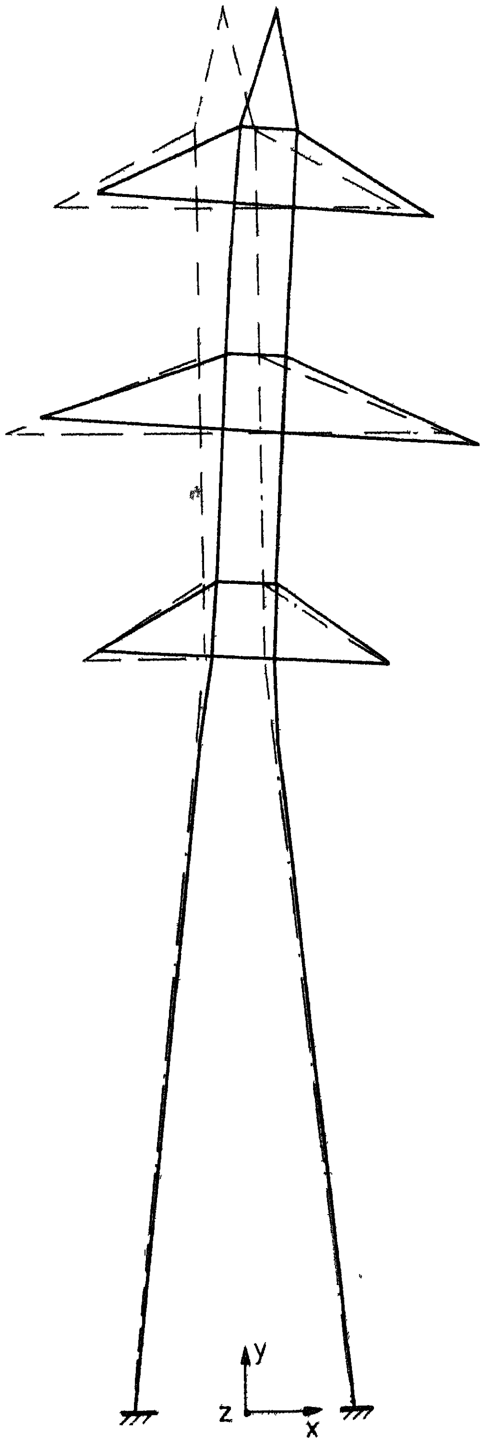
Iteration No.	Approximation to Eigenvalues of given problem, Eq. (5.17)					
	1	2	3	4	5	6
1	844.69	874.08	1619.36	7319.56	7962.87	9061.61
2	844.67	874.06	1617.44	7125.46	7957.30	9051.26
3	844.66	874.06	1617.42	7106.34	7957.21	9051.13
4	844.66	874.06	1617.42	7104.19	7957.20	9051.13
5	844.66	874.06	1617.42	7103.90	7957.21	9051.13
6	844.66	874.06	1617.42	7103.86	7957.20	9051.13

Using the subspace iteration method, eigenpairs of a transmission line tower are computed. The tower considered has 15m body height and 4.33m base width. There are 6 panels in the body. The results of eigenvalue iteration are presented in Table 5.2.

Table 5.2 Eigenvalues of Transmission Line Tower

Iteration No.	Eigenvalues					
	1	2	3	4	5	6
1	691.39	710.83	1230.85	6171.59	7046.35	8041.77
2	691.39	710.82	1230.15	6014.02	7042.86	8033.62
3	691.39	710.83	1230.15	6002.25	7042.82	8033.55
4	691.39	710.83	1230.15	6001.26	7042.81	8033.55
5	691.39	710.83	1230.15	6001.16	7042.81	8033.55
6	691.39	710.83	1230.15	6001.15	7042.81	8033.55

The mode shapes corresponding to the first six eigenvalues listed in Table 5.2 are sketched in Figs. 5.3(a) to (f). In these figures, the bending modes have components only in a plane, i.e., either in X-Y plane or Y-Z plane. The components in the perpendicular plane is negligibly small (of the order of 10^{-5} or less whereas those in the plane is of the order of 10^{-2}). This confirms that the eigenspectrum is a series of planar bending modes and torsional modes



(a) First mode

Fig.5.3 Mode shapes of the tower

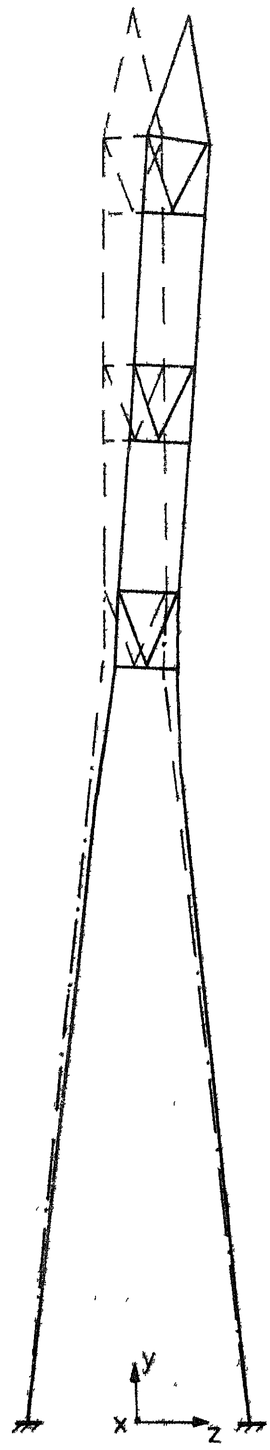


Fig.5·3 (b) Second mode

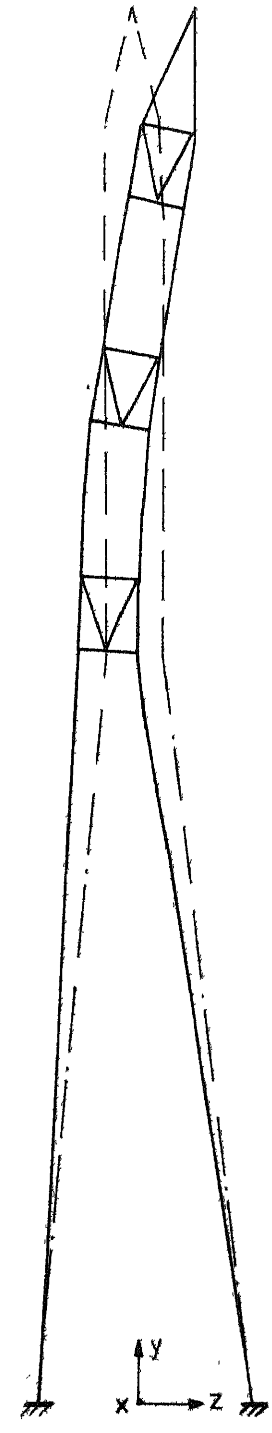


Fig. 5·3 (c) Sixth mode

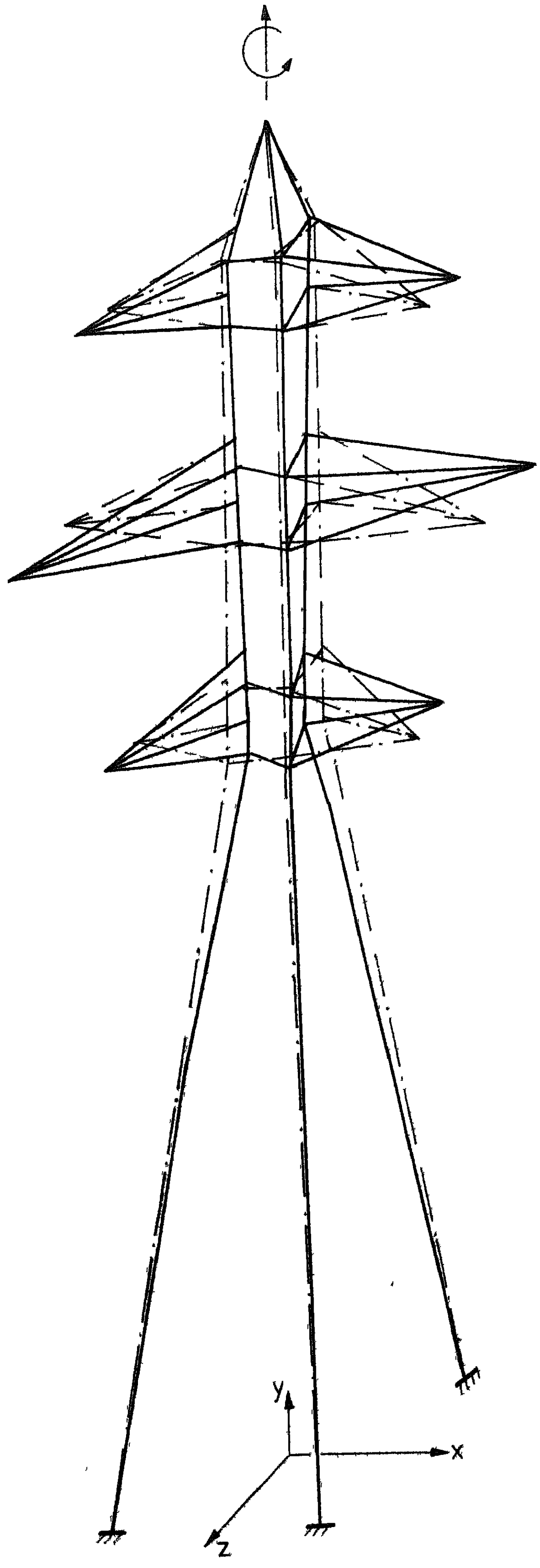


Fig. 5.3(d) Third mode

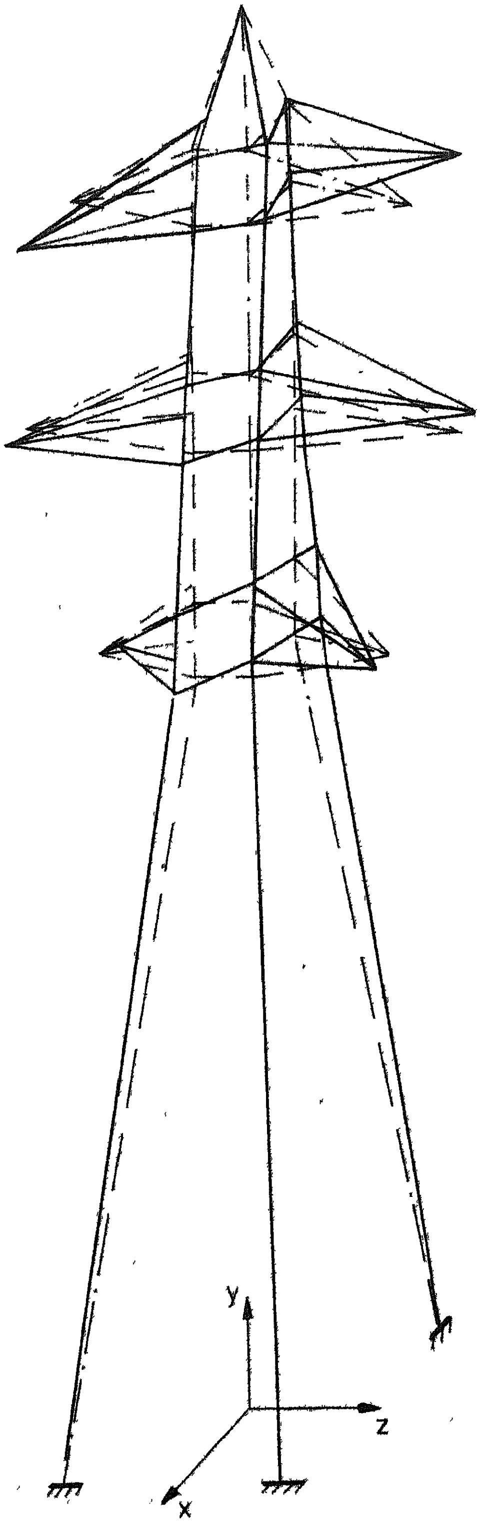


Fig.53(e) Fourth mode

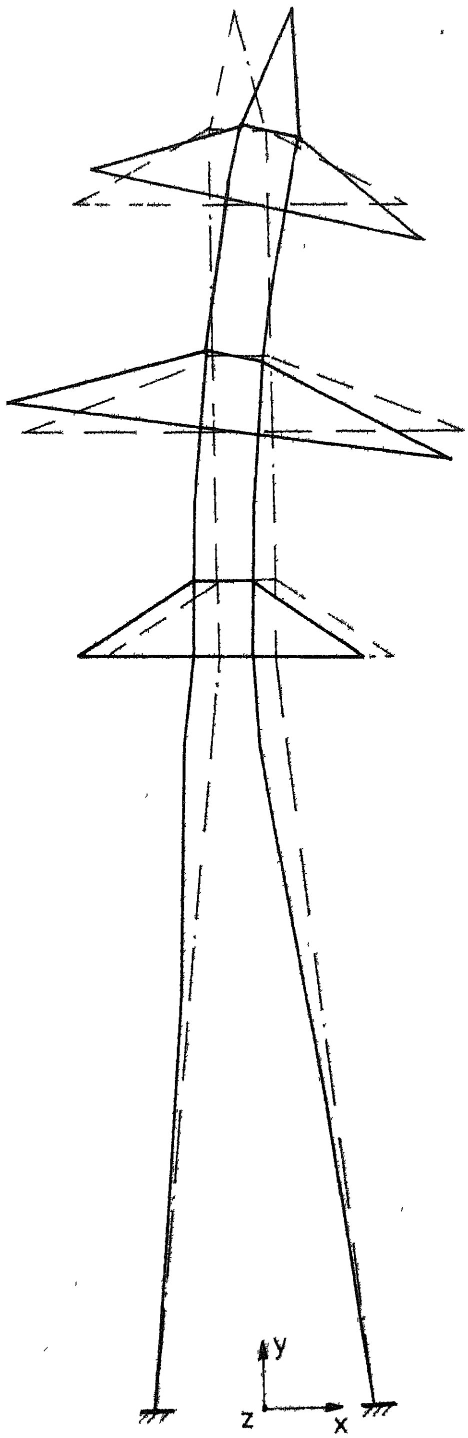


Fig.5.3(f) Fifth mode

starting from the lowest end of the spectrum.

The lowest eigenvalue λ_1 is 691.39 and hence the fundamental natural frequency $\omega_1=\sqrt{691.39} = 26.3$ rad/sec. The large values of ω_i are due to the fact that the elements of mass matrix are very small in comparison to the elements of the stiffness matrix.

The second eigenvalue is very close to that of the first. These two eigenvalues correspond to the first bending modes in the mutually perpendicular planes, Fig.5.3(a) and (b). The stiffness of the tower is almost the same in these two planes. The small difference is only due to the presence of cross arms in one of them and hence, the first two eigenvalues turns out to be close.

The two torsional modes lie adjacent to one another in the series. The eigenvalues of these modes are very well spaced. From the eigenvalues given in Table 5.2, it is observed that the first three eigenvalues, corresponding to the first mode in each of the principal directions, being close to each other can be called a set. The rest three eigenvalues belonging to the second set of eigenmodes are very well separated from the first three.

5.2.4 Parametric Study

A study on the variation of eigenvalues with height of tower has been conducted. Eigenvalues are extracted for

transmission line towers having total height of 28.0m, 30.5m, 33.0m, 35.5m and 38.0m. The height of basket is taken as 13m (Fig. 2.6) and is constant for all the towers considered in this study. The width of base has been fixed at 4.33m for all towers based on the optimum design results for static loads. The panel heights are also decided from the observations on the optimum design results, Chapter 4. The results of parametric study are presented in Table 5.3.

The base width being kept constant, the results presented in Table 5.3 are essentially a function of tower height. These results are plotted in Fig. 5.4(a) and Fig. 5.4(b) against the height of tower. The results are drawn in two separate figures, since ω_4^2 is about 10 times of ω_1^2, i.e., spaced much apart.

As discussed in the previous section, the first two eigenvalues are close to each other and the difference goes on diminishing for increasing height. Beyond certain height, say 40m (Fig. 5.4(a)), the eigenvalues ω_1^2 and ω_2^2 of the space transmission tower become equal, but the eigenmodes are different. This shows that the effect of cross arm on overall stiffness of the tower becomes negligible for high towers. The third eigenvalue ω_3^2, corresponding to torsional mode, is separated apart and this does not come close to other lower bending modes for all heights of

Table 5.3 Eigenvalues for Different Tower Heights

Total height of the tower (m)	Number of degrees of freedom	Eigenvalues extracted						CPU time* taken Min.,sec.
		1	2	3	4	5	6	
28.0	177	691.39	710.83	1230.15	6001.15	7042.81	8033.55	5, 15
30.5	189	558.52	570.83	994.73	5168.15	5789.13	6551.12	5, 18
33.0	201	448.12	455.85	822.94	4378.41	4806.59	5389.03	5, 51
35.5	213	364.16	369.14	711.18	3867.02	3994.02	4420.04	5, 48
38.0	225	299.11	302.43	630.87	3368.63	3379.29	3683.20	7, 05

* IBM 7044 computer

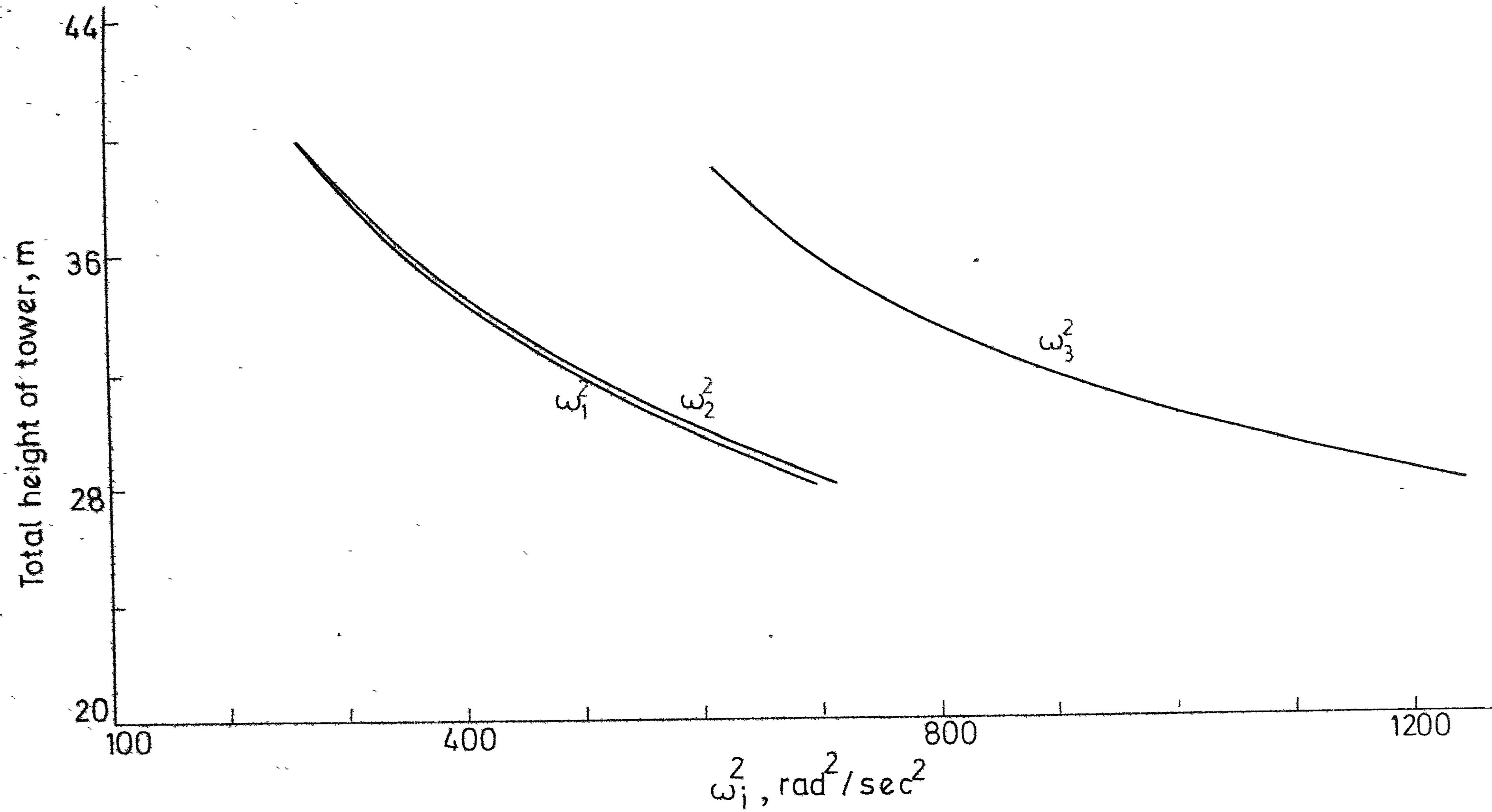


Fig.5·4(a) Height of tower versus ω_i^2 (i=1,2,3)

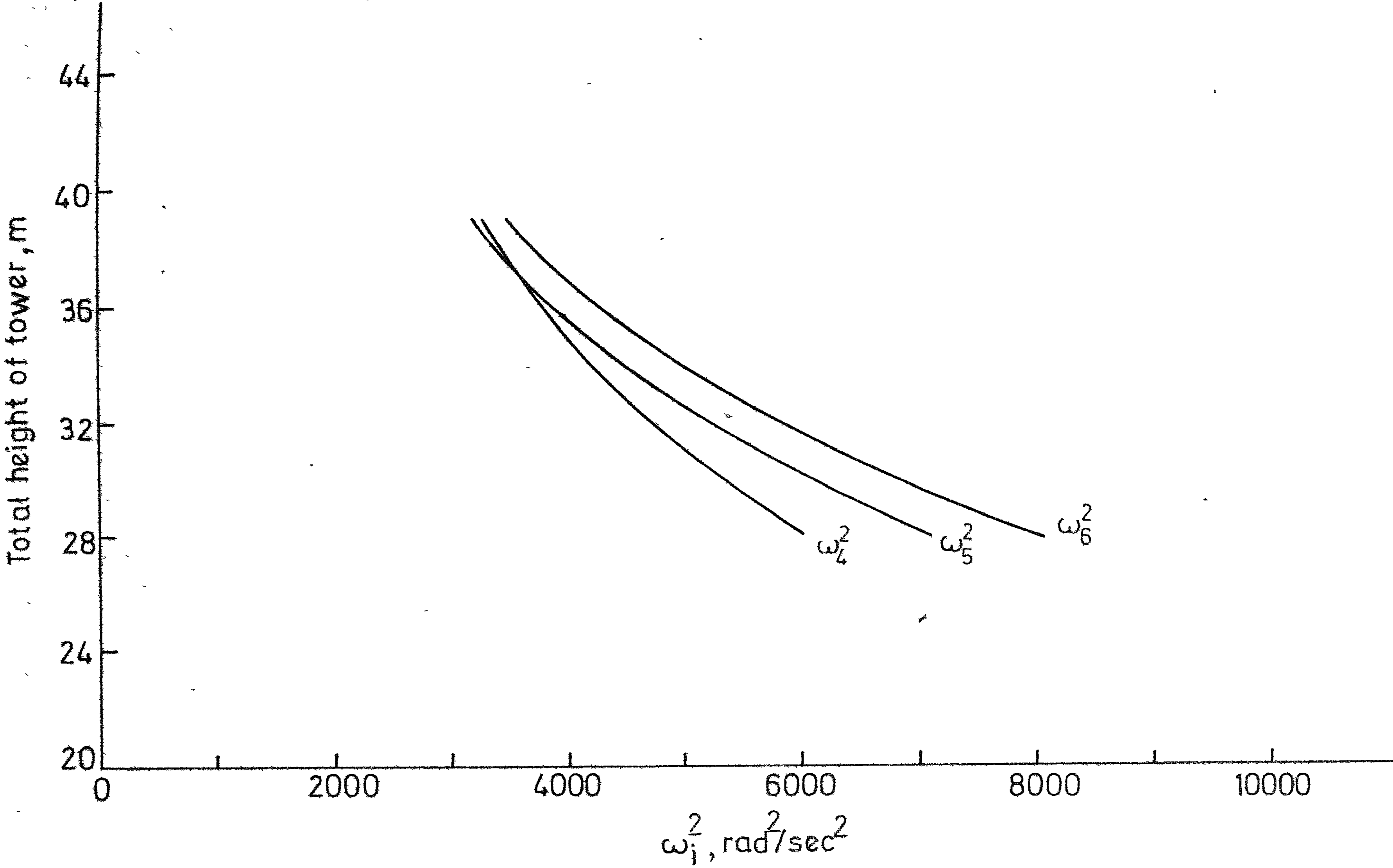


Fig. 5·4 (b) Height of tower versus ω_i^2 (i = 4, 5, 6)

transmission line towers considered.

In Fig. 5.4(b), ω_4^2 correspond to the second torsional mode, ω_5^2 to the second bending mode in X-Y plane and ω_6^2 to second bending mode in Y-Z plane. This order of eigenmodes remained the same (also listed in page 108) for heights upto 37m. The order gets changed for higher towers, i.e., for tower height greater than 37m, Fig. 5.4(b). In the changed spectrum, the fourth and fifth eigenmodes interchange their position. The absolute slope of ω_3^2 curve, Fig. 5.4(a), which corresponds to the first torsional mode, fast increases than that of ω_1^2 and ω_2^2 curves as the height increases. Furthermore, the absolute value of slope of the curve ω_4^2 is more than that of ω_5^2 and ω_6^2 for the range of heights considered. These observations show that the eigenvalues corresponding to bending modes approach zero faster than that of torsional modes as the height increases. Hence, for high towers, the torsional modes move away leaving the bending modes to dominate the lowest end of the eigenspectrum.

For tall towers, the torsional modes are driven away from the lowest end and hence for slender towers only bending modes need be extracted for a reasonably accurate dynamic analysis. Furthermore, as noted earlier, for slender towers, the eigenvalues corresponding to the

bending modes in two mutually perpendicular planes become equal. These two observations state that the modelling of structure can be further simplified and idealized as a plane frame. This boils down to the conventional dynamic analysis of multistoreyed buildings and television towers, wherein the adjacent mass points are idealized to lie in a vertical axis and are assumed connected in between by a single equivalent plane frame member.

5.3 Dynamic Response

5.3.1 Wind Load Analysis

The n_f uncoupled equations, Eq. (5.7), can be solved if the diagonal matrix, Δ , derived from damping matrix $\mathbf{C}$ is known. But in practical dynamic analysis the matrix $\mathbf{C}$ is not known. However it is usual practice to adopt an equivalent fraction of critical damping, for the structure considered. The same has been adopted in the present work. Therefore, the j-th equation of the uncoupled system, Eq. (5.7), i.e., j-th modal equation of motion, turns out as

$$\ddot{q}_j(t) + 2\beta_j \, \dot{q}_j(t) + \omega_j^2 \, q_j(t) = \sum_{i=1}^{n_f} \phi_{ij} \, p_i(t) \qquad (5.19)$$

where suffix i denotes i-th degree of freedom,

$\beta_j = \xi\omega_j$, ξ is the fraction of critical damping

ϕ_{ij} - i-th component of the j-th mode shape vector, which are M-orthonormal

$p_i(t)$ - load function at i-th degree of freedom.

Assuming initial conditions to be zero, the solution of Eq. (5.19) is given[88] as

$$q_j(t) = \sum_{i=1}^{n_f} \phi_{ij} \{ \frac{1}{\omega_{jd}} \int_0^t p_i(t)\ e^{-\beta j(t-\tau)} \sin \omega_{jd}(t-\tau)\ d\tau \} \tag{5.20}$$

where $\omega_{jd} = \sqrt{\omega_j^2 - \beta_j^2} = \omega_j^2 \sqrt{1-\xi^2}$.

The functions $p_i(t)$ can be replaced, for computational purposes, as a product,

$$p_i(t) = F_i\ f_i(t) \tag{5.21}$$

where F_i is a constant, i.e., maximum value of load $p_i(t)$ over the time interval considered. $f_i(t)$ is a function of time which takes a maximum value of $\pm$ 1.0. Substituting Eq. (5.21) in Eq. (5.20), the response equations turn out to be,

$$q_j(t) = \sum_{i=1}^{n_f} \phi_{ij}\ F_i \{ \frac{1}{\omega_{jd}} \int_0^t f_i(\tau)\ e^{-\beta j(t-\tau)} \sin \omega_{jd}(t-\tau)\ d\tau \} \tag{5.22}$$

Introducing the concept of dynamic load factor (DLF), which is defined as the ratio of dynamic displacement to static displacement, the maximum value of q_j — the modal participation factor, is given by

$$q_{j,max} = \sum_{i=1}^{n_f} \phi_{ij} \, F_i \, (DLF)_{ij,max} \Big/ \omega_j^2 \qquad (5.23)$$

where $(DLF)_{ij,max}$ is defined as the maximum DLF for the unit load $f_i(t)$ due to oscillation under natural frequency ω_j rad/sec.

The peak values of q_j are being known from Eq.(5.23), the upper bound on displacement can be computed by superimposing the modal contributions.

$$u_{i,max} = \sum_{j=1}^{n_f} |\phi_{ij} \, q_{j,max}| \qquad (5.24)$$

The summations in Eq. (5.20) through Eq. (5.24) shall go from 1 to s, only, when s out of n_f eigenpairs from the lowest end of the spectrum are used to get a reasonable approximate analysis. Hence, in the rest of this chapter, superposition of only first s modes are considered for presentation.

In order to compute the maximum stress at any point on the structure, we need to know its displacement mode. We have available to us two choices: (1) to use the upper bound displacement vector whose components are given by Eq. (5.24), or (2) to use the natural modes of the structure, $\vec{\phi}_j$, j = 1,2,....s and then superpose the resulting stresses corresponding to each natural mode in proportion to the peak modal participation factors, Eq.(5.23).

The use of the upper bound displacement vector seems unreasonable because its components represent an upper bound to the corresponding peak displacement and is merely an approximation (though reasonable) to the actual displacement of the structure. Moreover, its use may lead to a conservative estimate of stress at one point on the structure and simultaneously may result into a nonconservative estimate of the stress at some other point on the structure.

The use of natural modes as indicated in choice (2) seems more logical, since the structure in any case vibrates in its natural modes. We have, therefore, the upper bound stress on any member i as

$$\sigma_{i,max} = \sum_{j=1}^{s} |\sigma_{ij}\ q_{j,max}| \tag{5.25}$$

where σ_{ij} is the stress in member i due to mode shape j taken as deflection vector.

5.3.1.1 Dynamic Load Factor (DLF)

The wind loads on joints at peak level have been calculated and shown in Fig. 2.6. The time function, f(t), is shown in Fig. 5.5. The method of computing $(DLF)_{max}$ for this unit load-time function is described below.

Dynamic load factor (DLF) is given by, from Eq.(5.22), for any general load function, f(t) as:

$$DLF = \frac{\omega^2}{\omega_d} \int_o^t f(\tau)\, e^{-\beta(t-\tau)} \sin \omega_d(t-\tau)\, d\tau \qquad (5.26)$$

The above expression is nothing but the displacement function, $y(t)$, since static maximum displacement, y_{st}, of $f(t)$ is 1.0. Hence, without any loss of generality, we can write the above equation as

$$y(t) = \frac{\omega^2}{\omega_d} \int_o^t f(\tau)\, e^{-\beta(t-\tau)} \sin \omega_d(t-\tau)\, d\tau \qquad (5.27)$$

The deterministic wind load function, $f(t)$, shown in Fig. 5.5 has four distinct segments as follows:

$$\begin{array}{lll}
(1) & f(t) = p\,\dfrac{t}{t_d} \quad ; \quad t_d = t_1 & o \leq t \leq t_1 \\
(2) & f(t) = p \quad ; & t_1 \leq t \leq t_2 \qquad (5.28) \\
(3) & f(t) = p+(1.0-p)\dfrac{t}{t_d} \;; t_d = t_3 - t_2 \;; & t_2 \leq t \leq t_3 \\
(4) & f(t) = 1.0-(1.0-p)\dfrac{t}{t_d} \;; t_d = t_4 - t_3 \;; & t_3 \leq t \leq t_4
\end{array}$$

The rest of the load function is the cyclic repetition of load segments (2), (3) and (4) listed above. Substituting the above functions in Eq. (5.27), we get $y(t)$ for different regions as given below:

<u>Region 1</u>: $o \leq t \leq t_1$

$$y = \frac{\omega^2}{\omega_d} \int_o^t \frac{p\tau}{t_d}\, e^{-\beta(t-\tau)} \sin \omega_d(t-\tau)\, d\tau$$

$$= \frac{p}{t_d}\left[t - \frac{2\beta}{\omega^2} + \frac{e^{-\beta t}}{\omega^2} \left\{2\beta \cos \omega_d t + \left(\frac{\beta^2}{\omega_d} - \omega_d\right) \sin \omega_d t\right\}\right] \qquad (5.29)$$

$$\dot{y} = \frac{p}{t_d} [1 - e^{-\beta t} (\cos \omega_d t + \frac{\beta}{\omega_d} \sin \omega_d t)] \tag{5.30}$$

Region 2 : $t_1 \leq t \leq t_2$

$$y = e^{-\beta t} (\frac{\dot{y}_o + \beta y_o}{\omega_d} \sin \omega_d t + y_o \cos \omega_d t)$$

$$+ \frac{\omega^2}{\omega_d} \int_o^t p \; e^{-\beta(t-\tau)} \sin \omega_d (t-\tau) \; d\tau$$

$$= p + e^{-\beta t} [(y_o - p) \cos \omega_d t + \frac{\dot{y}_o + \beta y_o - \beta p}{\omega_d} \sin \omega_d t] \tag{5.31}$$

$$\dot{y} = e^{-\beta t} [\dot{y}_o \cos \omega_d t - \frac{\dot{y}_o \beta + \omega^2 (y_o - p)}{\omega_d} \sin \omega_d t] \tag{5.32}$$

where $y_o = y(t=t_1)$ from Eq. (5.29)

$\dot{y}_o = \dot{y} \; (t=t_1)$ from Eq. (5.30)

Region 3: $t_2 \leq t \leq t_3$; load increasing from p to 1.0

$$y = e^{-\beta t} (\frac{\dot{y}_o + \beta y_o}{\omega_d} \sin \omega_d t + y_o \cos \omega_d t)$$

$$+ \frac{\omega^2}{\omega_d} \int_o^t [p + (1.0 - p) \frac{\tau}{t_d}] \; e^{-\beta(t-\tau)} \sin \omega_d (t-\tau) \; d\tau$$

$$= e^{-\beta t} (\frac{\dot{y}_o + \beta y_o}{\omega_d} \sin \omega_d t + y_o \cos \omega_d t)$$

$$+ p [1 - e^{-\beta t} (\cos \omega_d t + \frac{\beta}{\omega_d} \sin \omega_d t)]$$

$$+ (\frac{1-p}{t_d}) [t - \frac{2\beta}{\omega^2} + \frac{e^{-\beta t}}{\omega^2} \{ 2\beta \cos \omega_d t + (\frac{\beta^2}{\omega_d} - \omega_d) \sin \omega_d t \}]$$

$$= p + \frac{1-p}{t_d} (t - \frac{2\beta}{\omega^2}) + e^{-\beta t} (A \cos \omega_d t + B \sin \omega_d t) \tag{5.33}$$

where $A = y_o - p + \frac{(1-p)\, 2\beta}{t_d \omega^2}$

$$B = \frac{\dot{y}_o + \beta y_o - p\beta}{\omega_d} + \frac{(1-p)\,(\beta^2 - \omega_d^2)}{\omega^2\, t_d\, \omega_d}$$

$y_o = y(t=t_2-t_1)$ from Eq. (5.31)

$\dot{y}_o = \dot{y}(t=t_2-t_1)$ from Eq.(5.32)

$$\dot{y} = \frac{1-p}{t_d} + e^{-\beta t}\, [(B\,\omega_d - \beta A) \cos \omega_d t - (\beta B + A\omega_d) \sin \omega_d t\,] \tag{5.34}$$

<u>Region 4:</u> $t_3 \leq t \leq t_4$; load decreasing from 1.0 to p

$$y = e^{-\beta t} \left(\frac{\dot{y}_o + \beta y_o}{\omega_d} \sin \omega_d t + y_o \cos \omega_d t\right)$$

$$+ \frac{\omega^2}{\omega_d} \int_0^t [1.0 - (1.0-p) \frac{\tau}{t_d}\,]\, e^{-\beta(t-\tau)} \sin \omega_d (t-\tau)\, d\tau$$

$$= 1.0 - \frac{(1-p)}{t_d} \cdot (t - \frac{2\beta}{\omega^2}) + e^{-\beta t} (A \cos \omega_d t + B \sin \omega_d t) \tag{5.35}$$

where $A = y_o - 1 - (\frac{1-p}{t_d})\ \frac{2\beta}{\omega^2}$

$$B = \frac{\dot{y}_o + \beta y_o - p\beta}{\omega_d} - \frac{(1-p)\,(\beta^2 - \omega_d^2)}{\omega^2\, t_d\, \omega_d}$$

$y_o = y\ (t = t_3 - t_2)$ from Eq. (5.33)

$\dot{y}_o = \dot{y}\ (t = t_3 - t_2)$ from Eq. (5.34)

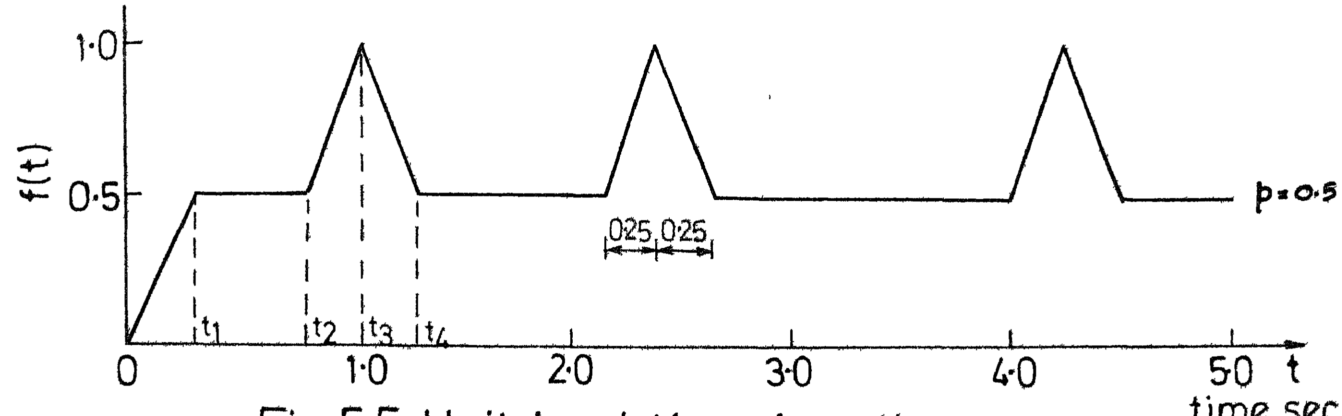


Fig. 5·5 Unit load-time function

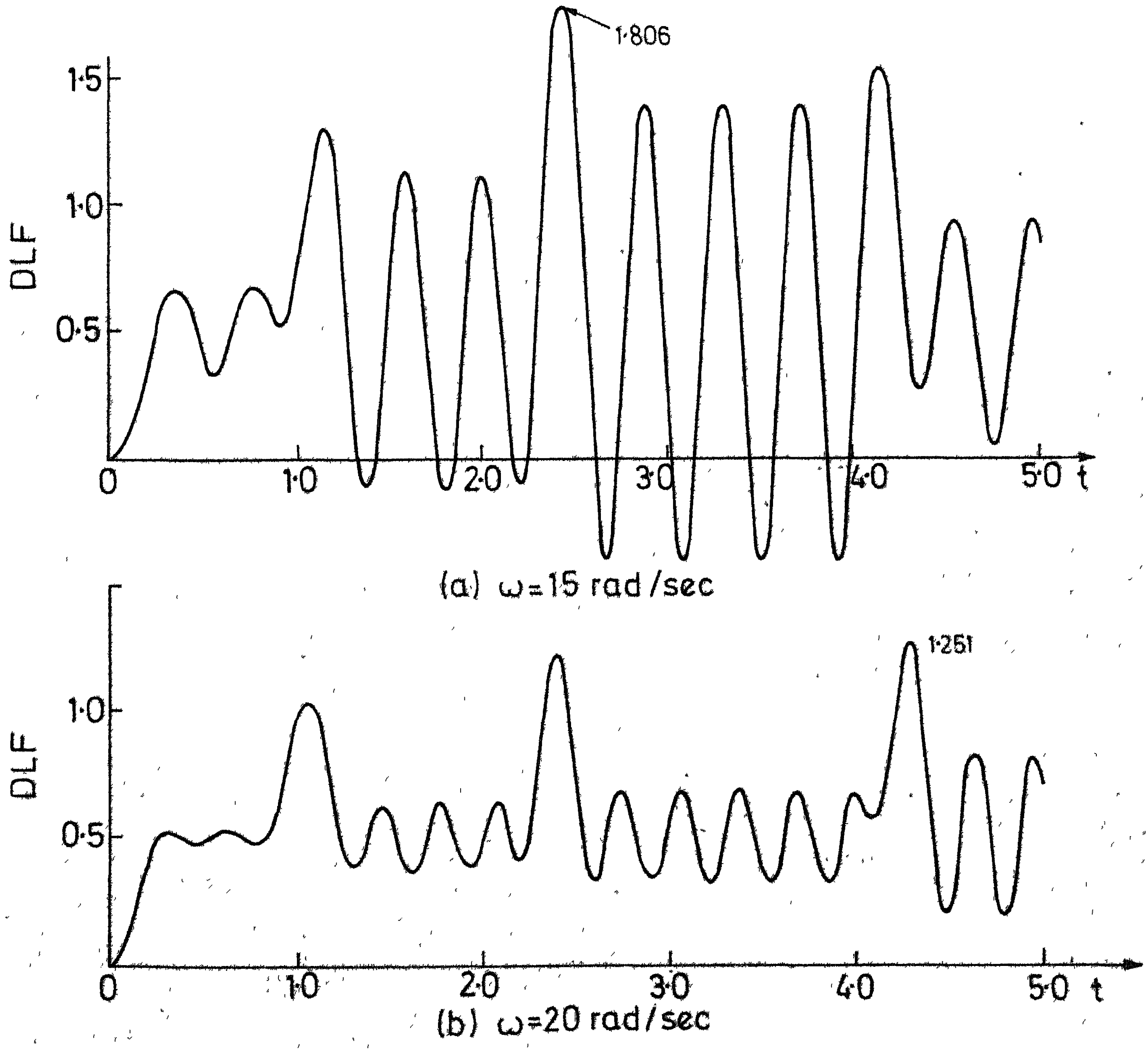
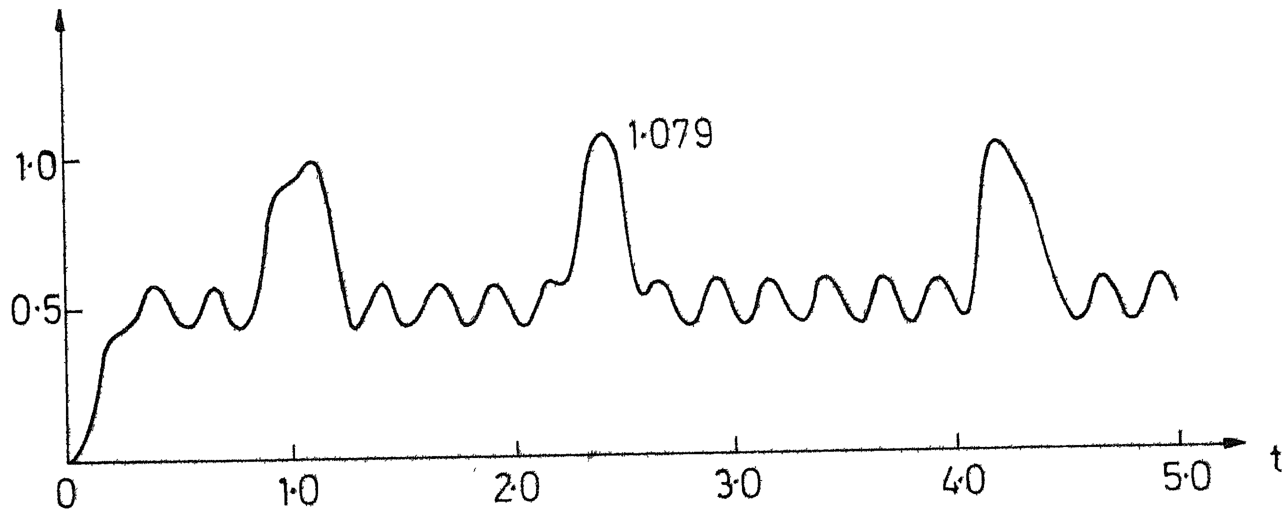
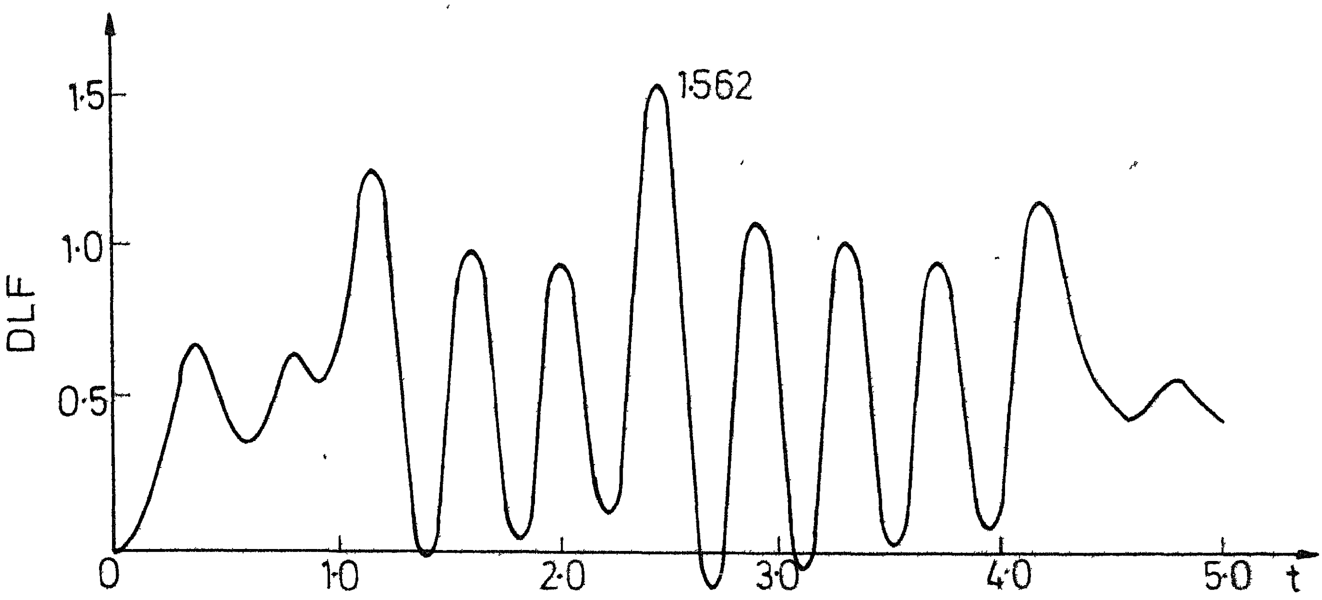


Fig. 5·6 DLF curves for undamped case

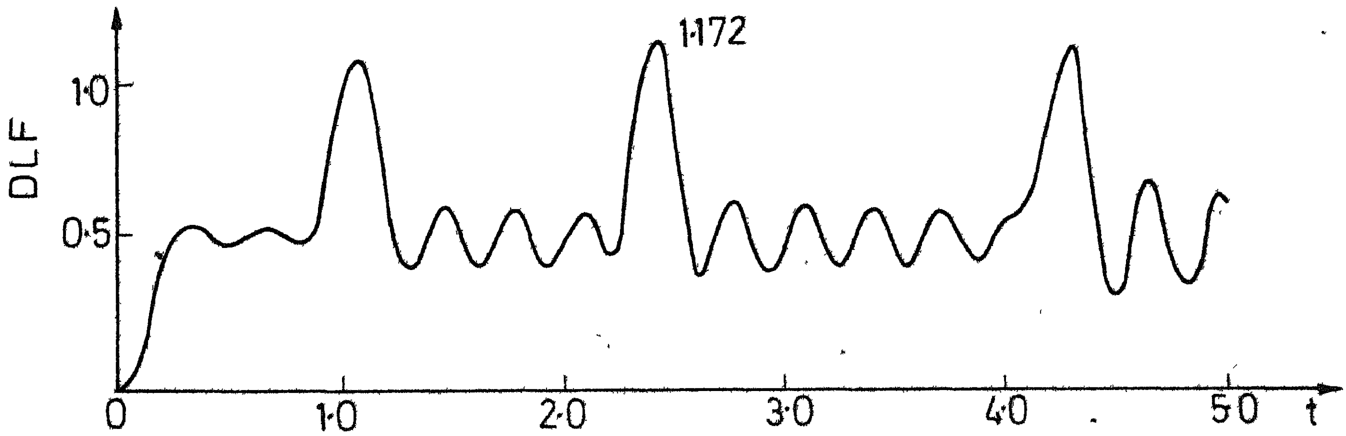


(c) $\omega = 25$ rad/sec

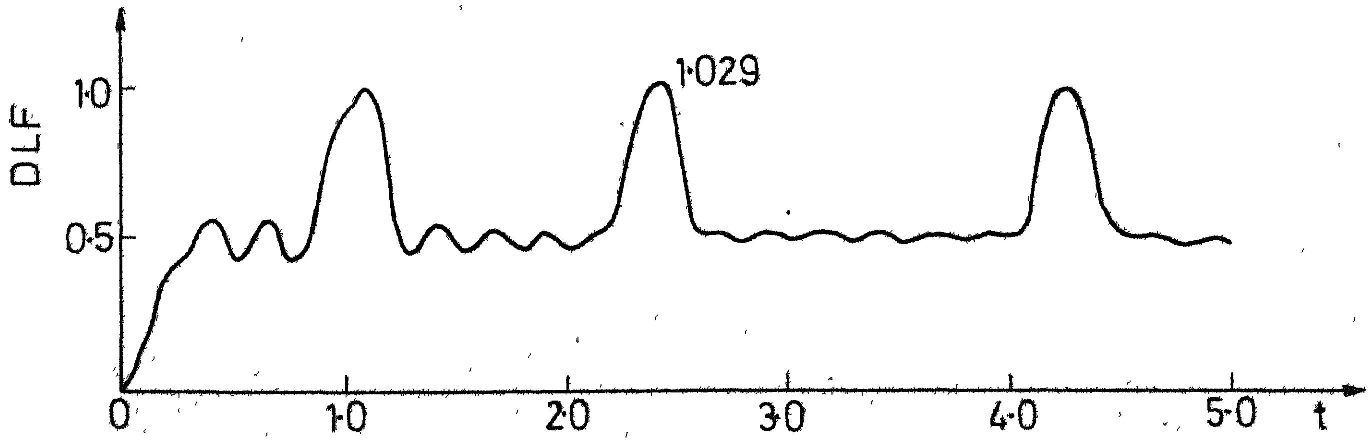
Fig. 5·6 (Continued)



(a) ω = 15 rad/sec



(b) ω = 20 rad/sec



(c) ω = 25 rad/sec

Fig.5·7 DLF curves for 2% damping

5.3.1.2 Maximum DLF

The curves in Figs. 5.6 and 5.7 show that the maximum value of DLF occurs during the period of gust. This suggest that the $(DLF)_{max}$ can be obtained by searching only that time interval during which gust appears. Since the maximum of y (or DLF) correspond to $\ddot{y} < 0$, Eq. (5.39), the length of time for which $\ddot{y} > 0$ can be ignored from such. Thus computations to obtain $(DLF)_{max}$ is considerably reduced. To get an overall variation of $(DLF)_{max}$ as a function of ω, $(DLF)_{max}$ is computed for ω ranging from 0.2 to 40 rad/sec at intervals of 0.2 rad/sec and the values are plotted in Fig. 5.8.

The response (or DLF) during gust is very much dependent on the initial conditions at the start of the gust. Furthermore, the function f(t) is multimodal. Hence, the $(DLF)_{max}$ curves in Fig. 5.8 are not monotic.

5.3.2 Earthquake Load Analysis

The j-th modal equation of motion of the system subject to earthquake load is given by

$$\ddot{q}_j(t) + 2\beta_j\,\dot{q}_j(t) + \omega_j^2\,q_j(t) = -\,\ddot{y}_s(t)\sum_{i=1}^{n_f}\phi_{ij}\,m_i \qquad (5.40)$$

where $\ddot{y}_s(t)$ is the prescribed support acceleration and q(t) is the relative modal displacement with respect to the

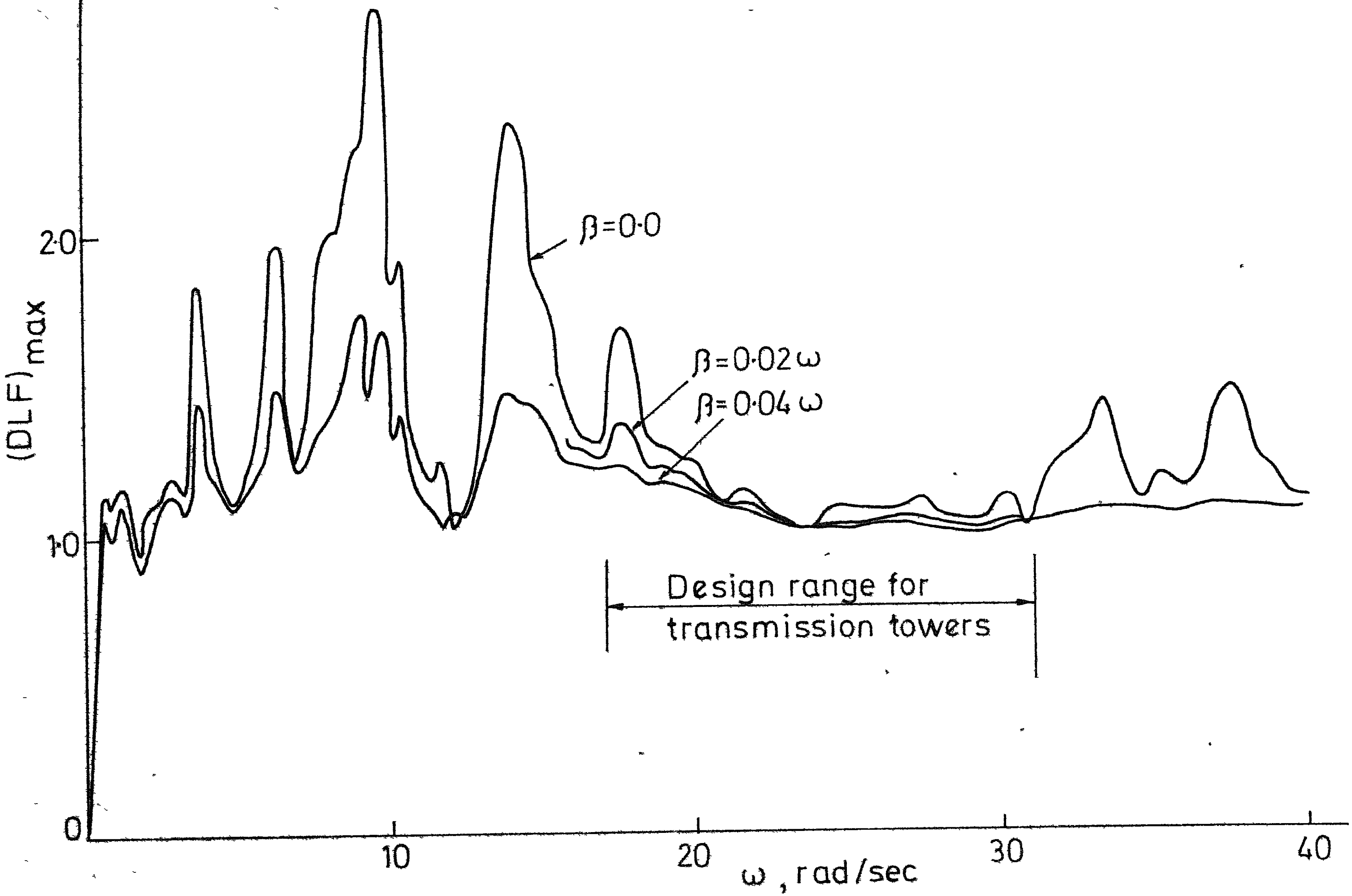


Fig. 5·8 $(DLF)_{max}$ versus ω

support. Proceeding similar to that of wind analysis, the maximum value of q_j can be obtained, for the present analysis, as

$$q_{j,max} = \frac{S_{a,j}}{\omega_j^2} \sum_{i=1}^{n_f} \phi_{ij} m_i \tag{5.41}$$

where $S_{a,j}$ is the acceleration spectrum value corresponding to the natural frequency ω_j rad/sec. From Eq. (2.2) we have

$$S_{a,j} = \alpha_h g = \beta I F_o S_a \tag{5.42}$$

The quantities in the above equation have been defined in Chapter 2 and the value of S_a is read from Fig. A1.1 for appropriate natural period and damping of the structure. The rest of the precedure for calculation of response of the structure follows exactly as in the case of wind load analysis.

5.4 Results of Dynamic Analysis

5.4.1 Wind Load Analysis Results

Transmission line towers having body heights of 15m, 20m, and 25m and a constant base width of 4.2m are analysed for time dependent wind load. Static loads simultaneously acting on the tower are treated as a special case of dynamic loads, i.e., $(DLF)_{max} = 1.0$ for all ω. The maximum dynamic stresses in the members of tower are computed by modal superposition method under three critical

loading conditions discussed in Chapter 2. The results are presented in Tables 5.4 to 5.6.

The first column in these tables gives the connectivity of the leg members of the tower from the top. The first seven members correspond to the leg members of the basket and the rest to the body of the tower. These have been taken as representative member forces for the purposes of present discussion. The second column gives the estimate of maximum dynamic force in the corresponding member by considering the superposition of only first three modes of vibration from the lowest end of the spectrum in dynamic analysis. The third column shows similar results considering six modes of vibration. The last column gives the corresponding forces due to static analysis of the tower, discussed in Chapter 4, considering maximum wind loads as static loads. The member forces in columns 2, 3 and 4 are suitably modified by the factor of safety discussed in Chapter 2 to get the design forces.

Comparison of values in columns 2 and 3 reveals that the member forces vary substantially in the leg members at the top while those at the bottom of the tower differ by about 10 percent. The dynamic forces in the leg members of the body of the tower seen to be converging faster than

those in the basket. Since the intent of the present work is to study the optimum configuration of the tower subject to dynamic wind loads and since the configuration of the basket portion is fixed from non-structural considerations, it is reasonable from the point of view of accuracy as well as computer time to carry out the dynamic analysis by modal superposition of only first six modes from the lowest end of the spectrum. Consideration of higher modes shall increase the member forces only marginally.

Once again from the comparison of results in columns 3 and 4 of these tables, it is observed that the member forces due to dynamic considerations are 9 to 33 percent more than the corresponding forces due to static considerations. The percentage increase in member forces is more for higher towers. This confirms the earlier observation that the equivalent static wind load analysis is more conservative for small towers as compared to high towers. Therefore, to get a realistic assessment of member forces in transmission line towers a rigorous dynamic analysis is warranted.

Table 5.4 Member Forces in 15m Body Tower (6 Pannels)

Joints of one of the leg members in each group (Fig. 2.7)	Dynamic analysis		Static analysis
	3 mode superposition	6 mode superposition	
2, 8	2.184	3.214	7.220
8, 12	5.648	7.194	10.166
12, 16	13.617	14.995	15.904
16, 22	17.313	19.106	20.855
22, 26	21.854	27.209	26.241
26, 30	28.290	35.671	34.325
30, 36	32.659	38.850	36.607
36, 40	40.405	44.764	43.555
40, 44	45.336	46.257	49.086
44, 48	47.034	48.375	50.289
48, 52	50.407	53.037	53.215
52, 56	51.929	55.841	53.939
56, 60	54.189	59.132	55.710

Forces in metric ton

Table 5.5 Member Forces in 20m Body Tower (8 Panels)

Joints of one of the leg members in each group	Dynamic analysis		Static analysis
	3 mode superposition	6 mode superposition	
2, 8	1.518	2.782	7.125
8, 12	4.850	6.535	10.166
12, 16	14.616	15.560	17.473
16, 22	17.674	19.420	22.098
22, 26	22.754	25.415	26.250
26, 30	28.467	34.235	34.317
30, 36	31.775	38.504	36.744
36, 40	41.087	47.450	45.267
40, 44	44.829	49.450	48.370
44, 48	49.226	50.096	52.013
48, 52	54.790	55.111	56.544
52, 56	58.642	60.291	58.817
56, 60	62.858	65.848	61.594
60, 64*	65.939	70.311	63.159
64, 68	69.187	74.782	65.186

Forces in metric ton

* Extend the joint numbering beyond 63 in the same sequence of Fig. 2.7.

Table 5.6 Member Forces in 25m Body Tower (10 Panels)

Joints of one of the leg members in each group	Dynamic analysis		Static analysis
	3 mode superposition	6 mode superposition	
2, 8	1.050	2.536	7.048
8, 12	4.372	6.552	10.166
12, 16	15.571	17.174	18.691
16, 22	18.223	21.044	23.039
22, 26	24.005	27.088	26.261
26, 30	32.214	35.859	34.308
30, 36	33.953	39.458	36.902
36, 40	43.165	50.290	45.847
40, 44	49.948	56.142	51.194
44, 48	53.036	57.623	52.428
48, 52	59.864	62.336	58.200
52, 56	65.138	66.045	61.115
56, 60	71.293	72.118	64.862
60, 64	75.819	78.651	66.842
64, 68	80.862	85.669	69.501
68, 72*	84.607	91.392	71.011
72, 76	88.552	97.084	73.051

Forces in metric ton

* Extend joint numbering similar to 20m body tower

5.4.2 Earthquake Load Analysis Results

Based on the recommendation of IS:1893, wind load is not considered during earthquake. However other loads, viz., dead loads due to the conductors and ground wire are considered.

Ground acceleration due to earthquake is considered acting in the transverse direction (X direction) of the transmission line tower. Hence the ground acceleration in the other two directions, i.e., Y and Z directions, are zero. This is incorporated in the three dimensional dynamic analysis by making necessary changes in the calculation of $q_{j,max}$, Eq. (5.41). In other words, the summation in Eq. (5.41) is carried out for $i=1,4,7,\ldots,(n_f-2)$ only.

In the eigenvalue analysis, we have observed that the lumped masses at joints are very small. This suggests that the loading due to earthquake ground motion would be small. Hence as a first attempt, analysis considering loads only due to ground motion is carried out. This study gives the stress in the members due to earthquake load only. Of course the stresses due to the other loads can be superimposed.

Maximum member forces due to earthquake load are computed by considering the contribution from the first 6 modes. However, in this case the contribution due to the bending modes in the longitudinal direction as well as due to torsional modes is negligibly small.

A tower of 25m body height with 10 panels in the body and base width of 4.2m is considered. The member forces computed are given in Table 5.7, assuming the tower to be located in the most critical earthquake zone.

Table 5.7 Member Forces due to Earthquake Load

Joint numbers* of one of the leg members in each group (body portion)	Member force (ton)
36, 40	0.9271
40, 44	1.0046
44, 48	0.9936
48, 52	1.0298
52, 56	1.0110
56, 60	1.1050
60, 64	1.2059
64, 68	1.3100
68, 72	1.3975
72, 76	1.4824

* Same as in Table 5.6

From Table 5.7, it is clear that the members are subjected to stresses of very low magnitude. Hence it is concluded that earthquake loads are not critical for the design of transmission line towers.

5.5 Results of Optimization and Discussion

The optimum configuration design obtained under static loads and mentioned in Chapter 4 is taken as starting design for the present study of optimization under dynamic loads. Available I.S. angles are used for the members of the tower. Member selection is made to satisfy the design criteria as per I.S. 802-1973, discussed earlier in Section 4.3. The preassigned parameters for the optimum design under static load condition (page 82) are kept same for this study too.

Optimum design is carried out for 15m, 20m, and 25m heights of tower body. The results of optimum design are presented in Table 5.8 and Table 5.9 respectively for undamped system and for 2 percent damping. The CPU time taken for one function evaluation is also given in Table 5.8. The weight of the tower at the end of each iteration is plotted in Figs.5.9 and 5.10 for undamped case and for 2 percent damping respectively.

From Table 5.8, it is observed that the optimum base width varies from 4.34m to 5.49m when the tower is

Table 5.8 Results of Optimization Study Under Dynamic Loads (Undamped Case)

	15m body, 6 panels		20m body, 8 panels		25m body, 10 panels	
	Starting Design (S.D)	Optimum Design (O.D)	S.D	O.D	S.D	O.D
Panel heights(m)	2.00	1.85	1.90	1.90	2.10	2.10
	2.20	2.20	2.10	2.10	2.20	1.58
	2.40	2.40	2.30	2.30	2.30	1.90
	2.60	2.60	2.50	2.30	2.40	2.40
	2.80	2.37	2.50	2.35	2.50	2.56
	3.00	3.58	2.70	2.70	2.50	2.45
			2.90	2.90	2.60	2.59
			3.10	3.45	2.70	3.00
					2.80	2.75
					2.90	3.67
Base width(m)	4.00	4.34	4.00	4.60	4.00	5.49
Lowest natural frequency, ω_1	27.756	28.926	22.348	23.349	18.033	20.679
Total weight(kg)	3766.447	3672.028	4823.799	4714.947	7043.425	6034.063
Reduction in weight (percent)	2.50		2.26		14.35	
CPU*time for one function evaluation(sec)	6.83		7.85		11.04	

* DEC 1090 computer which is about 10 times faster compared to IBM 7044

Table 5.9 Results of Optimization Study Under Dynamic Loads (2 percemt damping)

	15m body, 6 panels		20m body,8 panels		25m body, 10 panels	
	S.D	O.D	S.D	O.D	S.D	O.D
Panel heights(m)	2.00	1.10	1.90	1.90	2.10	1.31
	2.20	1.30	2.10	2.10	2.20	1.84
	2.40	2.10	2.30	2.30	2.30	2.31
	2.60	2.72	2.50	2.50	2.40	2.40
	2.80	2.80	2.50	2.50	2.50	2.50
	3.00	3.98	2.70	2.70	2.50	2.50
			2.90	2.90	2.60	2.45
			3.10	3.10	2.70	3.00
					2.80	2.65
					2.90	4.04
Base width(m)	4.00	4.02	4.00	4.30	4.00	5.20
Lowest natural frequency, ω_1	27.289	27.291	21.916	22.879	17.959	20.315
Total weight(kg.)	3688.956	3630.854	4823.799	4716.258	6598.937	5924.535
Reduction in weight (percent)	1.57		2.23		10.2	

assumed undamped. However, the range of optimum base width is 4.02m to 5.20m when 2 percent damping is considered, Table 5.9. Since 2 percent damping is recommended for steel structures, the latter values of optimum base width are more realistic for transmission line towers. Contrary to the results of optimum design under static loads, there is considerable variation in the base width corresponding to the optimum tower under dynamic loads. Optimum base width increases for increasing height under dynamic loads while it remained more or less constant under static loads. Thus, for a more realistic optimum configuration design of transmission line towers, considerations in the dynamic response regime seem to be inevitable.

Conclusions already made, from the study leading to optimum configuration under static loads, with respect to panel heights of transmission line towers remain unchanged under dynamic loading; i.e., the number of panels should be chosen such that the height of central panel(s) is 2.5m; the top and bottom most panel heights may be taken as 2.00m and 3.00m respectively; and the other panel heights may be fixed from linear interpolation of the top and bottom panel heights.

The reduction in weight is maximum corresponding to change in base width. This can be observed from Figs. 5.9 and 5.10 wherein the reduction in weight in the first

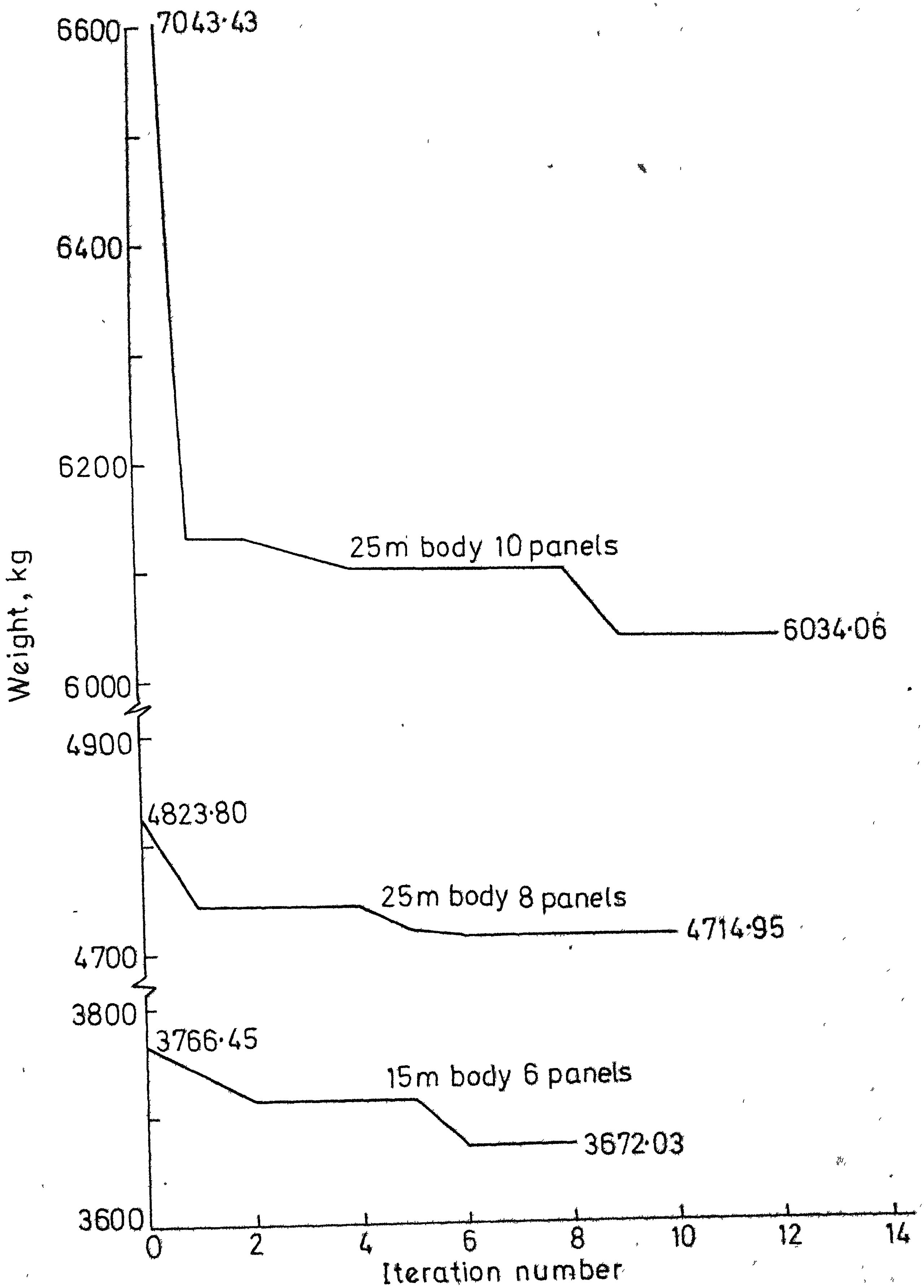


Fig. 5.9 Weight versus iteration (undamped)

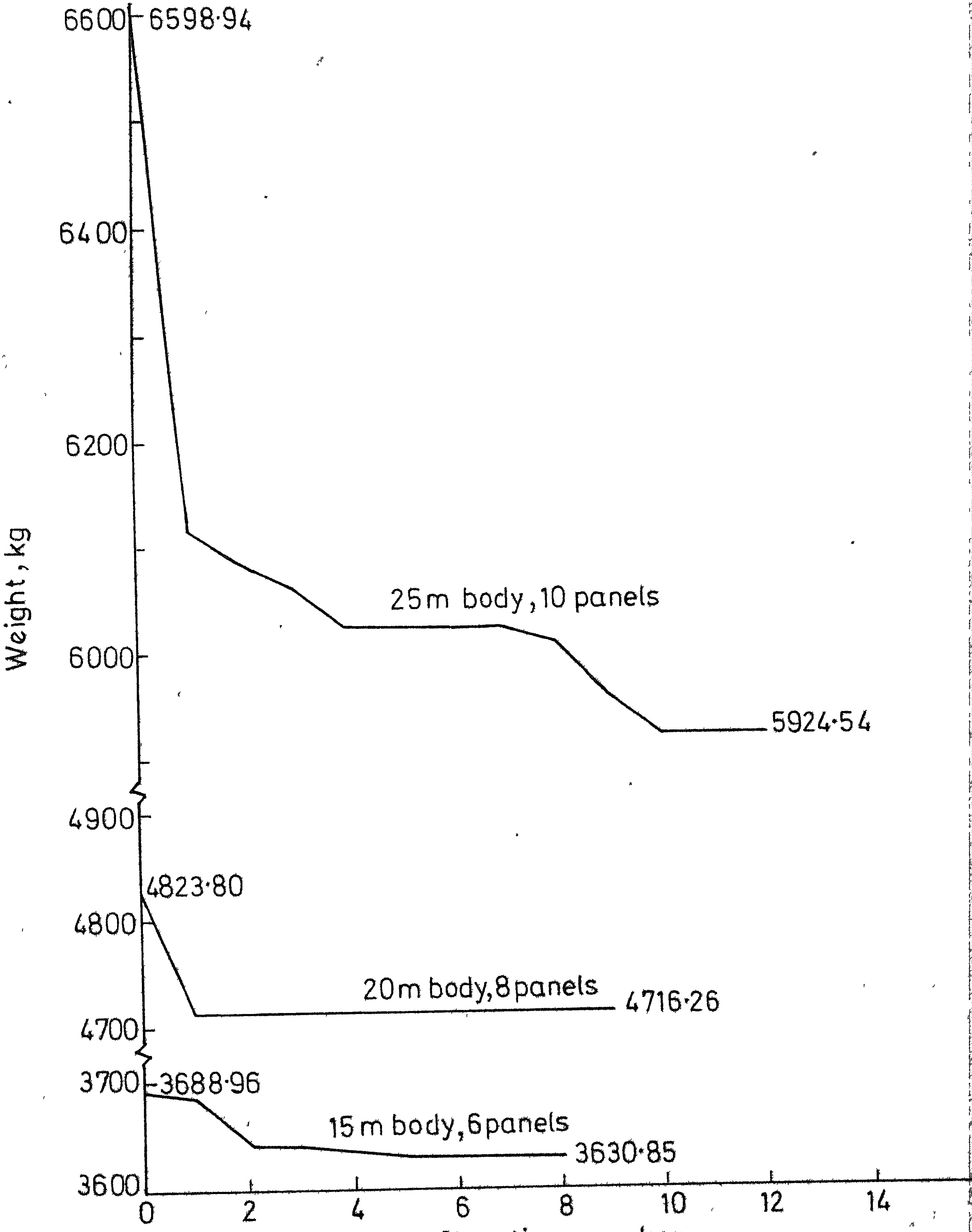


Fig.5·10 Weight versus iteration (2% damping)

iteration (which corresponds to change in base width) is significant. The reduction in weight due to change in panel heights (other design variables in the present formulation) is relatively small.

The variation of ω_i in the process of optimization is plotted in Fig. 5.11(a) to (f) for the six eigenvalues considered in the present work. In each plot, the corresponding ω_i is plotted for undamped and with 2 percent damping of three towers considered. From these graphs it is observed that the change (increase) in the value of natural frequencies is maximum in the first iteration, i.e., corresponding to change in the base width of the tower. Furthermore, changes in ω_i are negligible for the corresponding changes in panel heights. In other words, small changes in panel heights resulting during process of optimization do not significantly affect the natural frequency. Since the change in panel heights during the process of optimization is small and the corresponding changes in natural frequencies of the tower are also negligible, the total computational work involved can be substantially reduced as follows: The optimum design considering the base width and panel heights may be carried out first for static loads and then one-dimensional minimization only with respect to the base width be

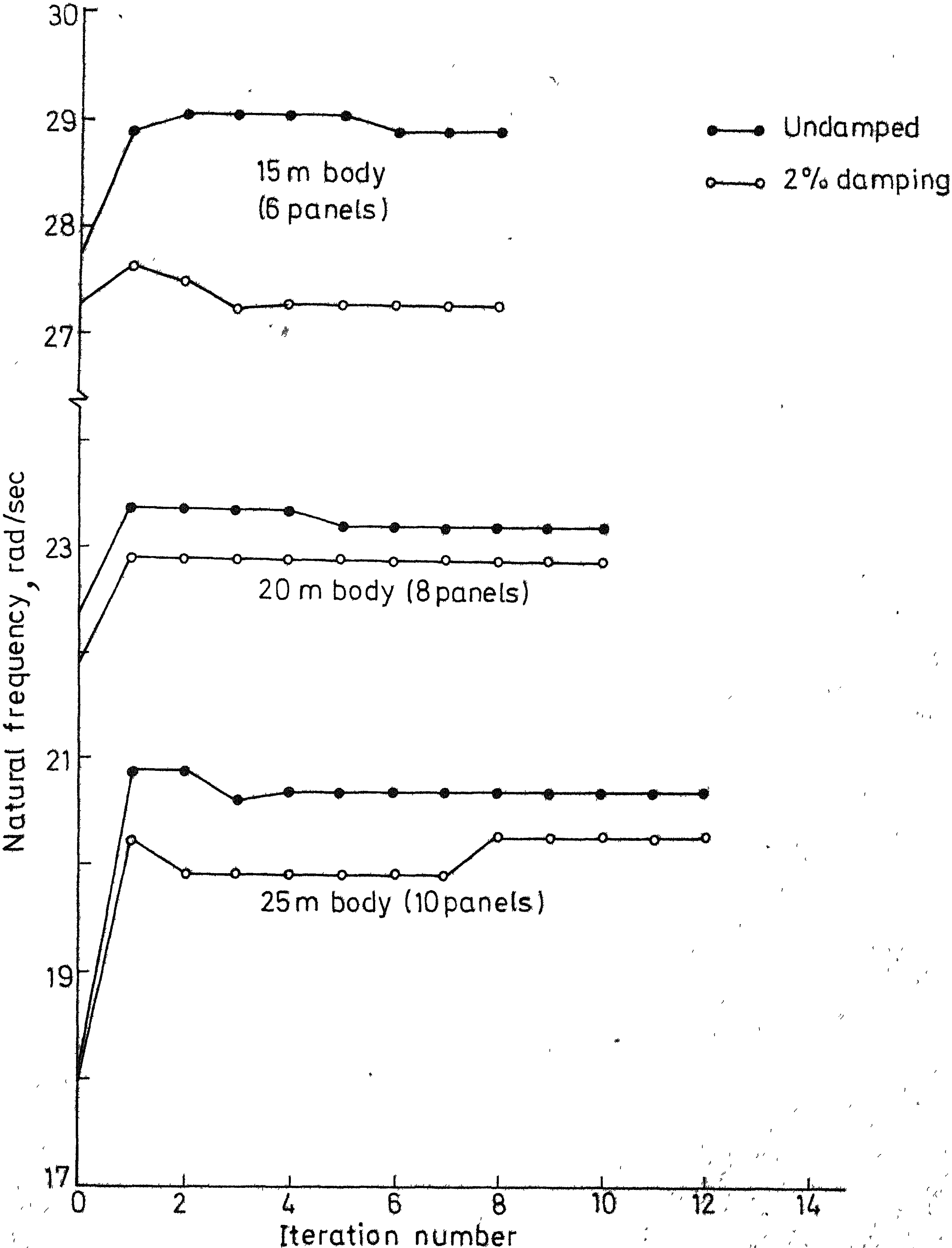


Fig.5·11(a) Variation of ω_1 during optimization

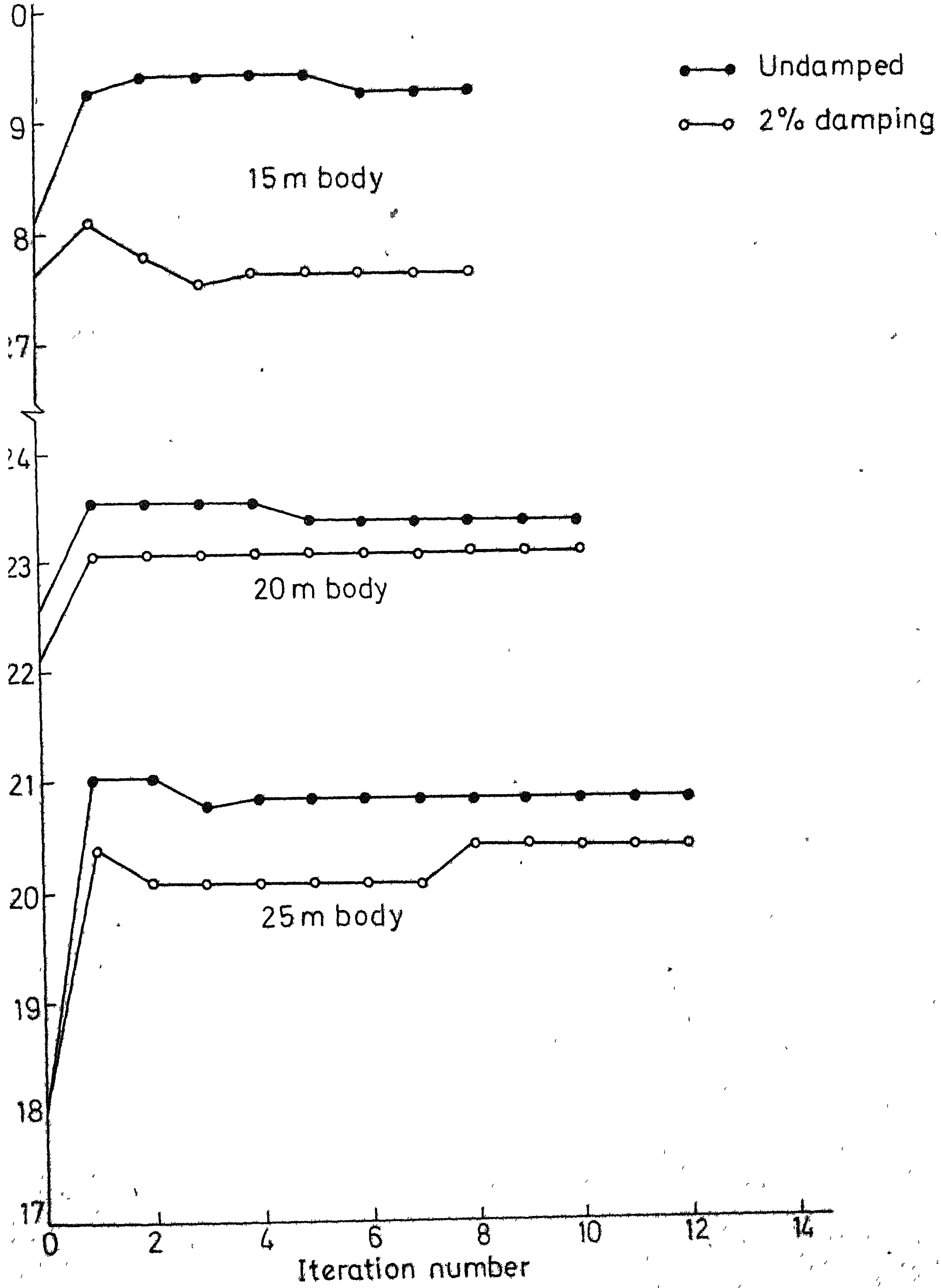


Fig.5.11 (b) Variation of ω_2 during optimization

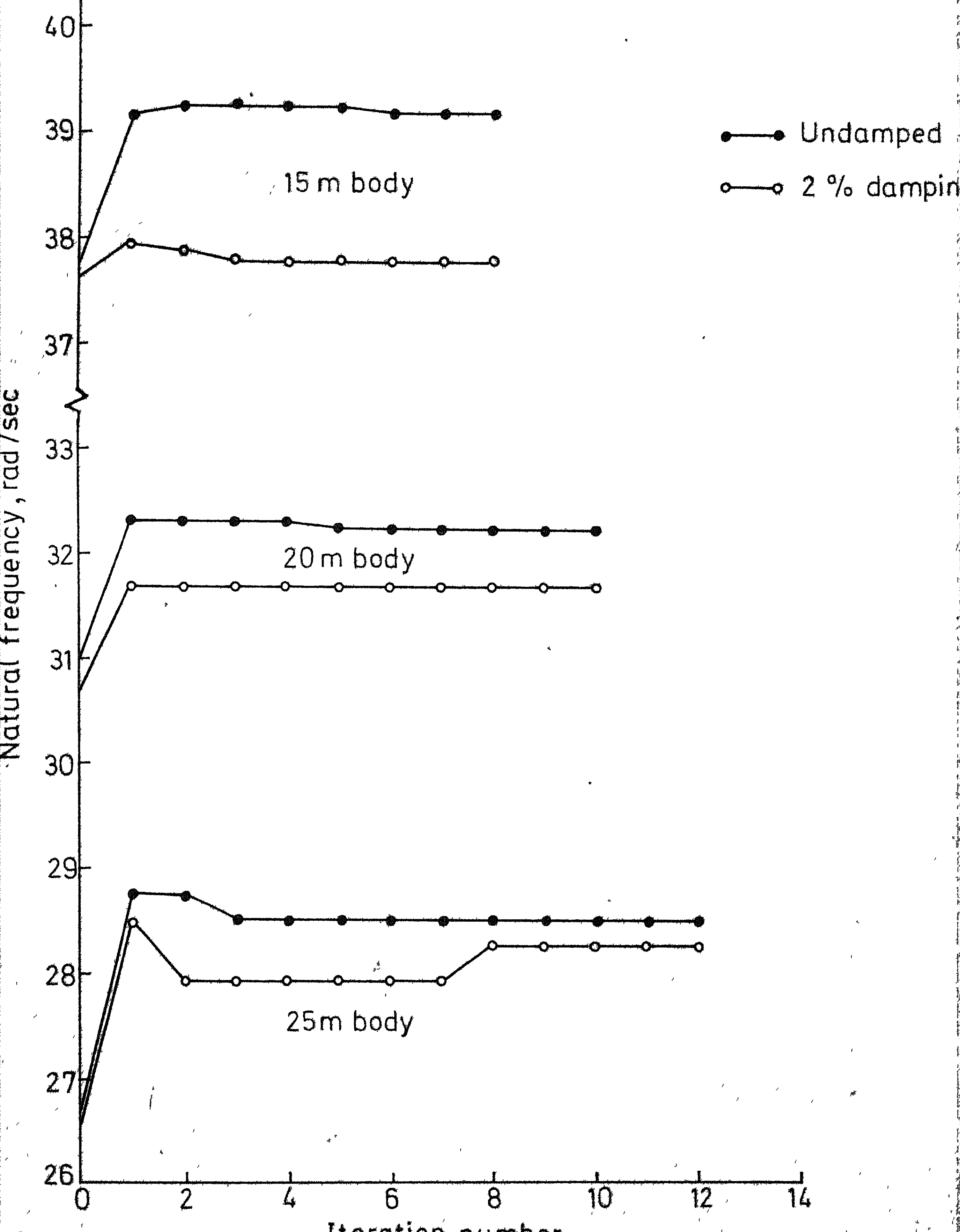


Fig.5·11(c) Variation of ω_3 during optimization

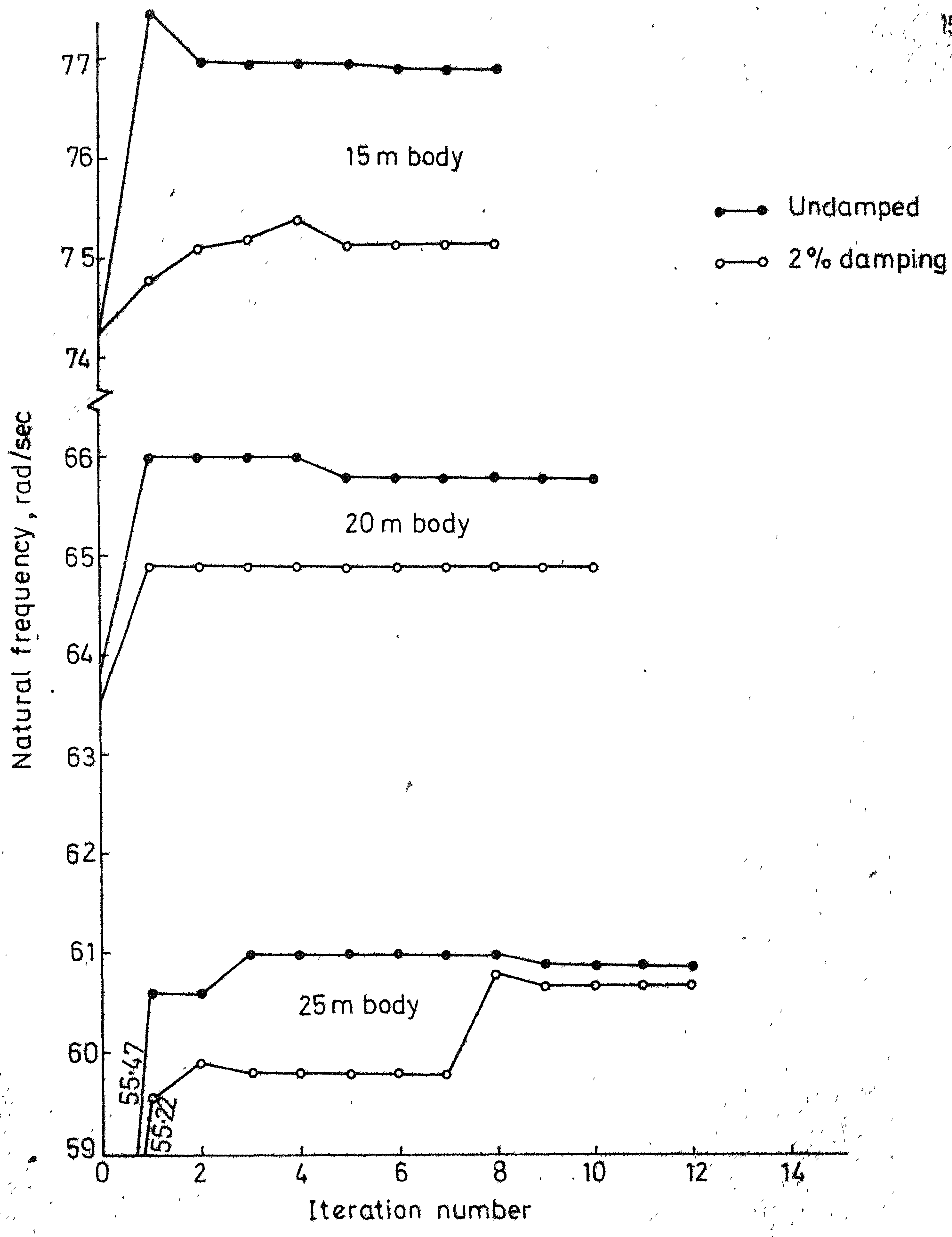


Fig.5·11(d) Variation of ω_4 during optimization

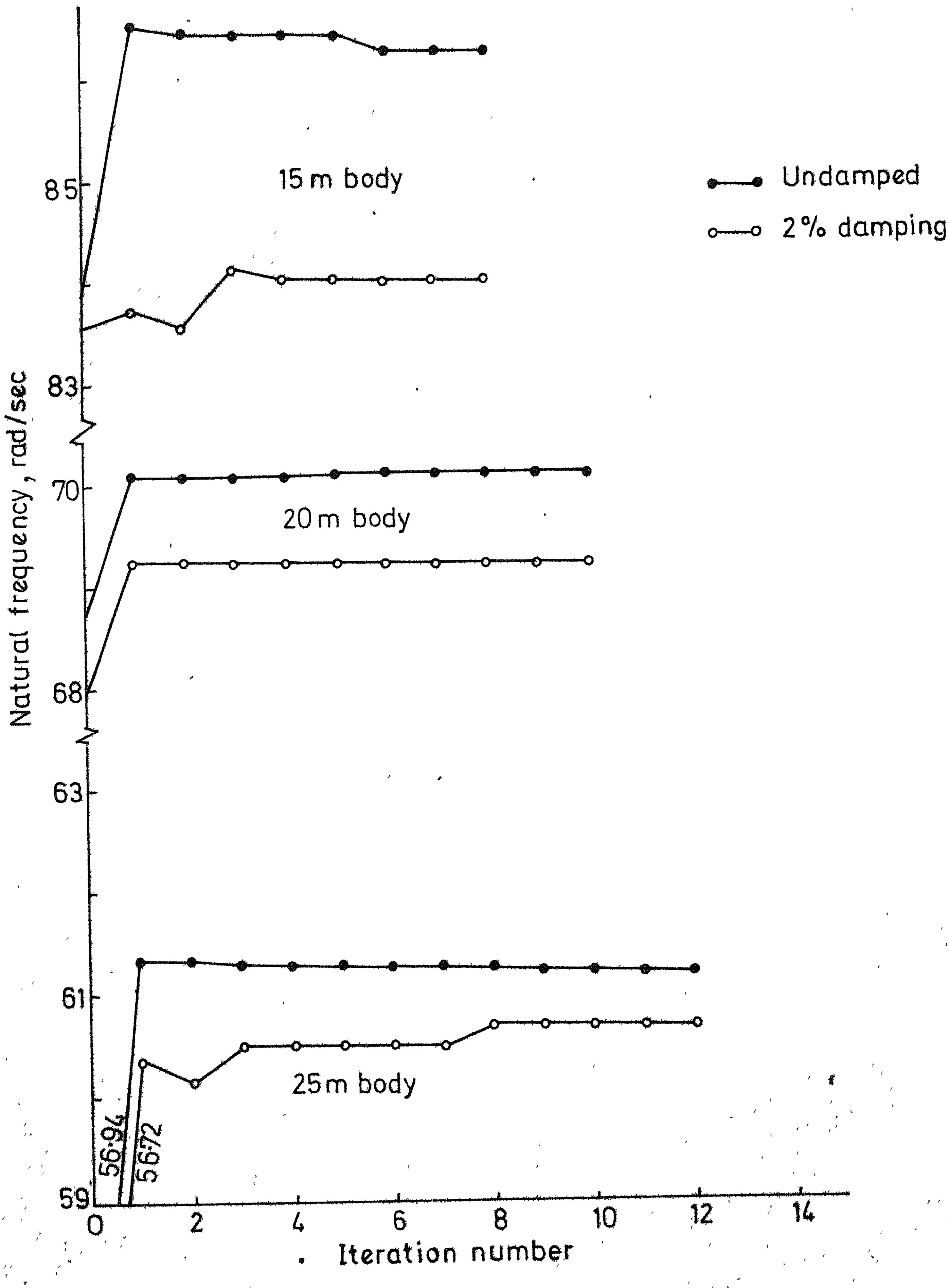


Fig. 5·11 (e) Variation of ω_5 during optimization

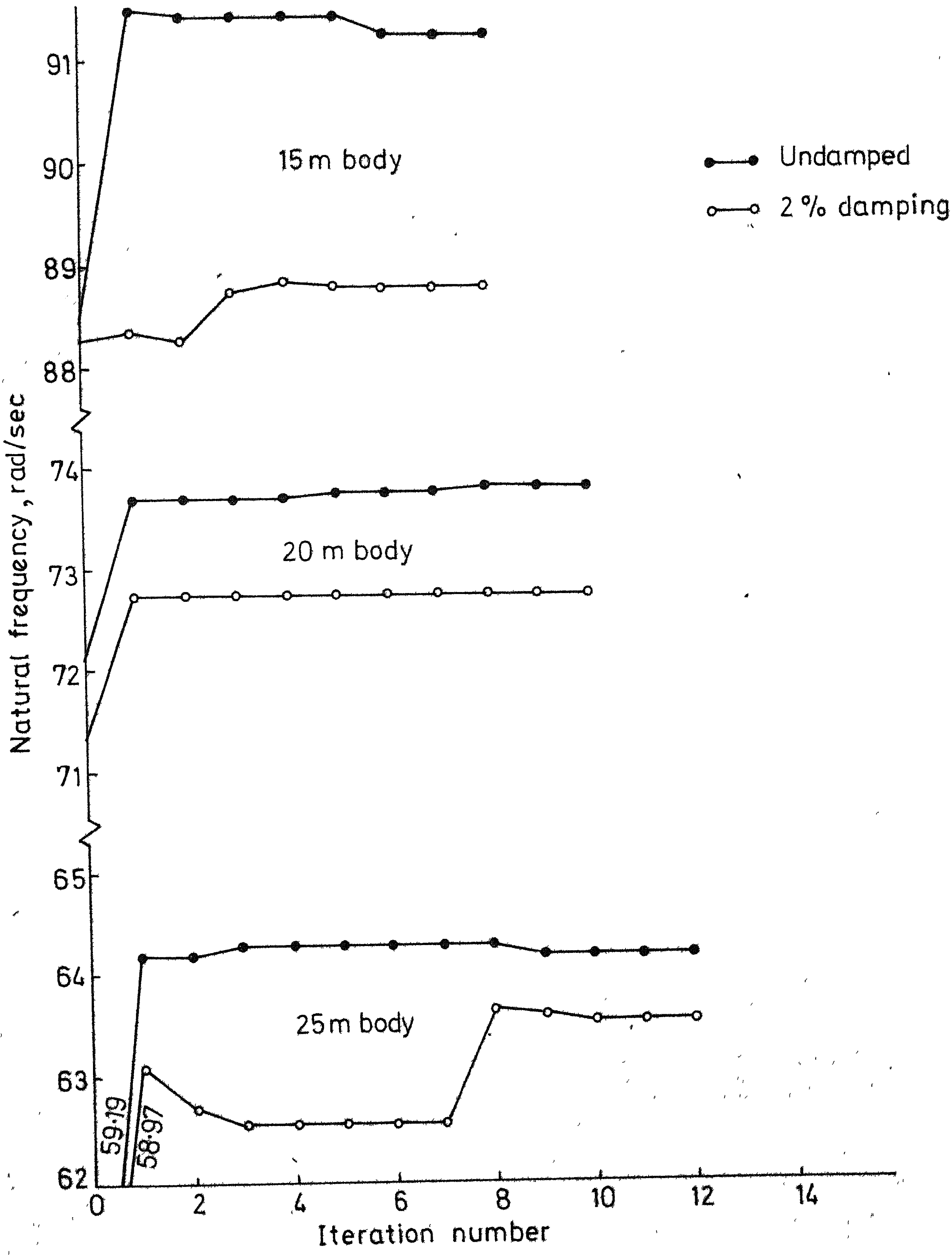


Fig. 5·11 (f) Variation of ω_F during optimization

undertaken in the dynamic response regime considering six eigenvalues from the lowest end of the spectrum.

In the present study, the change in natural frequency is an increase from the natural frequency of the corresponding starting design for all the transmission line towers considered, i.e., the tower becomes stiffer. This states that there is a reduction in the deflection at the joints. Hence it is justified from this observation that the constraints on deflection are not necessary for the optimum design of transmission line towers.

The area of leg members in the starting design as well as in the optimum design is plotted in Fig.5.12(a) to (c) against midheight of panels above ground. The reduction in the area of leg members is maximum in the body portion of the 25m body tower. In other words a considerable reduction takes place in the mass of the body of 25m body tower. On the other hand, there is maximum increase in the base width of this tower, Table 5.9. These two observations state that the optimum tower resist the dynamic loading by increasing the stiffness contribution and reducing the contribution due to inertia.

A parametric study has been carried out to observe the effect of damping on the optimum design. For this purpose, range of damping of 0 to 5 percent is considered.

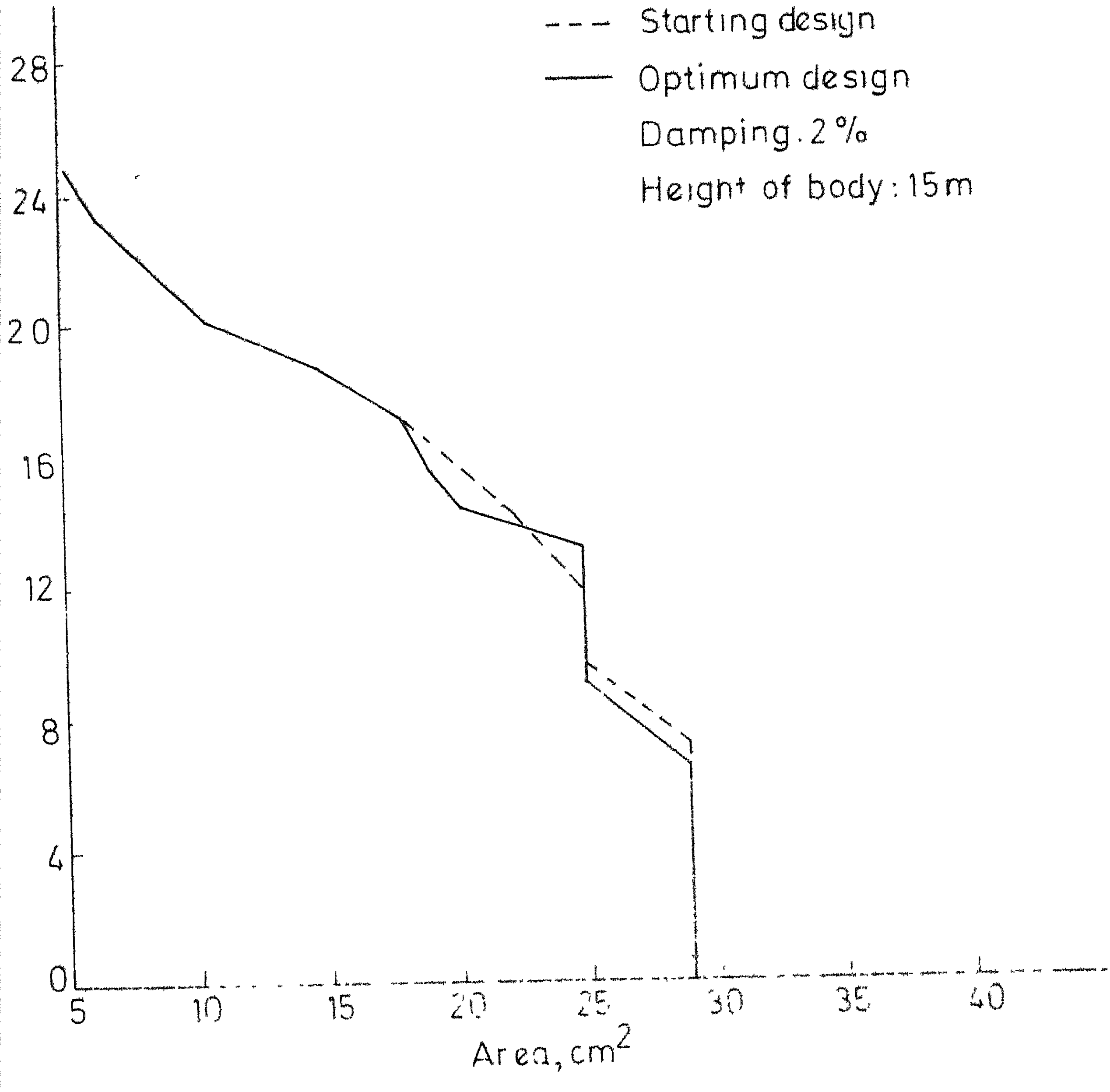


Fig.5·12(a) Area of leg members along the height of towe

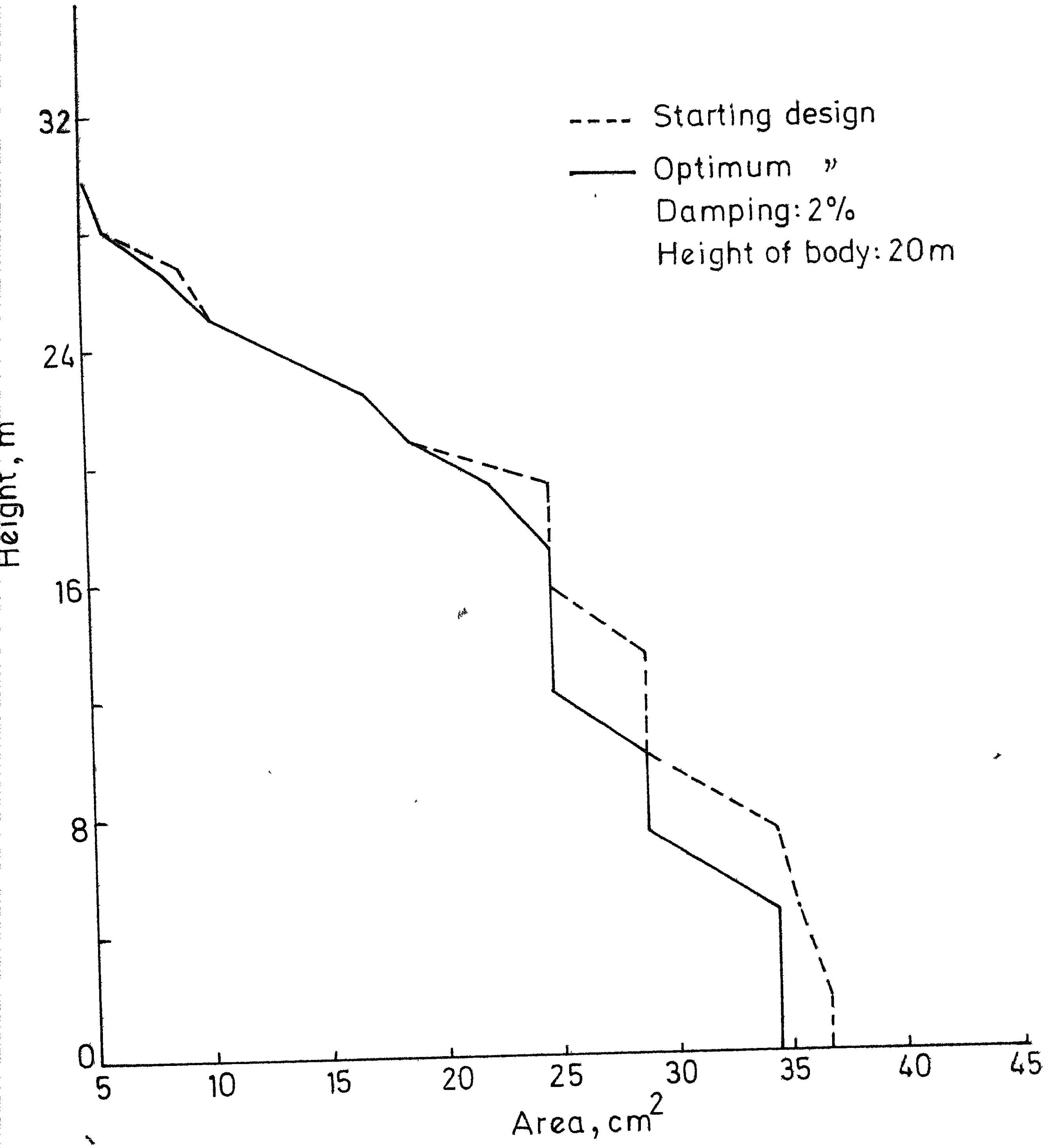


Fig. 5·12 (b) Area of leg members along the height of tower

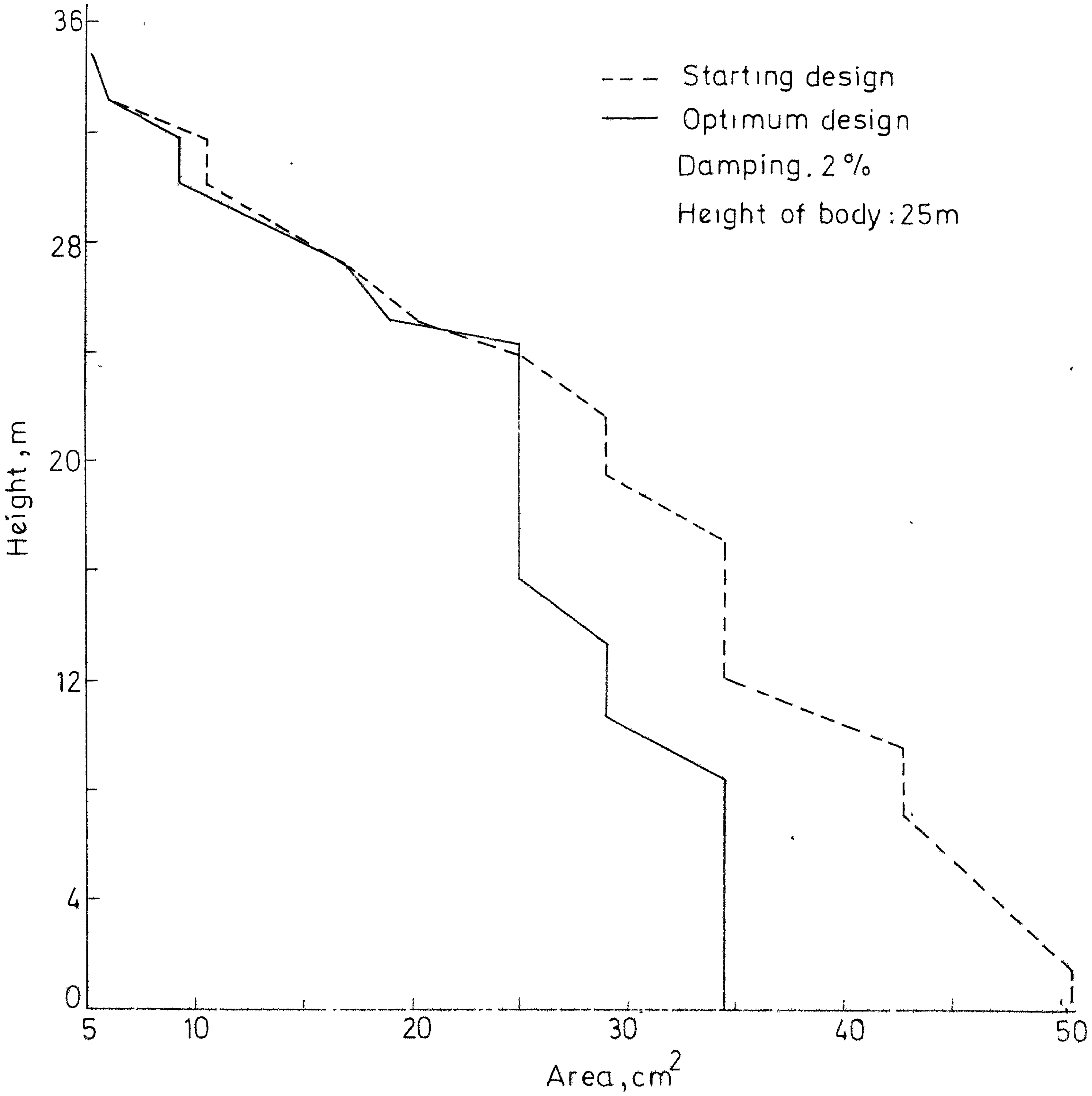


Fig. 5·12 (c) Area of leg members along the height of tower

The optimum weights obtained with different damping ratios are presented in Table 5.10. The corresponding optimum base widths are also given in this table.

From the results of parametric study presented in Table 5.10, it is observed that there is a general tendency of reduction in weight with increased damping. However, the reduction in weight is not much. This is due to the fact that the difference in $(DLF)_{max}$ (Fig. 5.8) is almost negligible for the range of natural frequencies corresponding to the optimum design of transmission line towers. From the results presented in Table 5.10, it is further noted that the reduction in weight is not monotonic with increased damping. This is explainable again from the $(DLF)_{max}$ curves plotted in Fig. 5.8 which itself is not a strictly monotonic function of ω for the ranges of ω under discussion. Furthermore, the discrete choice of final available sections as members of the tower also affects monotonicity of the convergence.

Table 5.10 Results of Optimum Design with Different Damping Ratios*

Sl.No.	Damping ratio (percent)	15 m body		20m body		25m body	
		Optimum base width (m)	Optimum weight(kg.)	Optimum base wdith (m)	Optimum weight(kg.)	Optimum base width (m)	Optimum weight(kg.)
1	0	4.34	3674.028	4.60	4714.947	5.49	6034.063
2	1	4.02	3616.830	4.30	4716.258	5.50	6048.663
3	2	4.02	3630.854	4.30	4716.258	5.20	5924.535
4	3	3.95	3668.007	4.56	4712.519	5.41	6036.838
5	4	3.94	3617.080	3.94	4789.293	5.43	6024.327
6	5	3.94	3593.230	4.28	4717.968	5.39	6009.267

* Starting base width has been taken as 4.00m for all designs presented in this table.

CHAPTER 6

CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

6.1 Introduction

The configuration optimization of transmission line towers has been carried out both for static and dynamic behaviour of the tower. The configuration design variables considered are the base width of the tower and the panel heights of tower body. The available angle sections have been used for the members of the tower. The resulting optimum configuration design problem turns out to be an unconstrained minimization problem which has been solved by using Powell's method.

The conclusions drawn from the results of the present study are grouped in Section 6.2. A set of recommendations for extension of the present work is given in Section 6.3.

6.2 Conclusions

(1) It is sufficient to model the transmission line towers as a space truss both for static and dynamic analysis. This conclusion for dynamic loads is based on the comparison of the eigenvalues of the towers and that of the individual members of the corresponding tower. This study establishes

that the response of the overall tower structure precedes that of individual members in the present case because the individual members are constrained to be designed for a l/r ratio of maximum of 200. This maintains the natural frequency of longest individual member by and large constant for towers of all sizes where as the frequency of the tower goes on decreasing for increasing tower height.

(2) The three critical loading conditions on the tower are the broken conditions of the ground wire, top conductor, and the middle conductor which need be considered for the purposes of design of transmission line towers. Earthquake loads are not critical for the design of transmission line towers.

(3) Stiffness approach is the most efficient method for the analysis of transmission line towers from the computational view point.

(4) Inorder to have a realistic response prediction for wind load, considerations of equivalent static loads are insufficient and a rigorous dynamic analysis is inevitable. This is more so when optimum design is the goal.

(5) A check, particularly for small towers, for safe response is warranted by the study of eigenvalues of the overall tower and of the longest member therein. This is due to the fact that in normal practice a conservative design

for small towers shall be produced by providing a wide base resulting in large stiffness of the tower. This may result in the overall natural frequencies of the tower higher than the natural frequency of the longest member in such towers. In that situation the dynamic response of the longest member shall govern the safe design.

(6) Transmission line towers are structurally quite stable and hence it is not necessary to embed overall stability considerations in the optimum design process.

(7) The variation in base width has marked effect on the weight of the tower. The optimum weight is more sensitive to this design variable in the dynamic response regime as compared to the static behaviour. From the results, it is observed that the base widths corresponding to optimum design tower for 15m, 20m, and 25m body heights are around 4.00m, 4.30m, and 5.40m respectively. In the optimum design under static loads these values turned out to be around 4.00m for all towers considered. The latter phenomenon in itself is not explainable. However, it gets corrected in the dynamic response regime.

(8) The number of panels in the body of the tower should be chosen such that the height of central panel(s) of the body is around 2.5m. Further, the top most panel

height may be kept around 2.00m and that of bottom most panel around 3.00m for achieving minimum weight tower. The intermediate panel heights may be fixed by linear interpolation of heights of top and bottom panel.

(9) The panel heights corresponding to optimum design of the tower turns out to be the same both under static and dynamic behaviour. Hence the panel heights may be decided from the optimum design under static loads. This leaves only a one-dimensional optimization to be performed with respect to the base width under dynamic loads so that overall computational time is minimized and realistic optimum design of transmission tower is achieved.

(10) The reduction in weight of tower for increase in damping is negligible.

(11) The conventional model of plane frame for the dynamic analysis of slender towers is confirmed to be reasonably accurate through the present study. However, since slenderness is only a relative concept, if the loading is such as to cause rotation about the vertical axis, then space truss model for dynamic analysis as concluded in (1) above will have to be adopted.

(12) The available Indian Standard angle sections are not rational, particularly the heavier ones from the view point of optimum design. These can be revised (proposal presented

in Appendix 2) so as to make them more rational from the point of view of minimum weight design of such structures.

6.3 Scope for Further Work

The present work has been carried out for 220 KV transmission line towers. The same can be extended for 440 KV and higher transmission voltages. This may require towers of very different configurations.

Wind loading is a random phenomenon. Therefore, it should be treated as random load to get more accurate response calculations and hence more rational optimum design. The random vibration analysis, of course, requires statistical data on wind over a reasonable period of time which is to be collected.

It has been observed that broken wire conditions occur very rarely, and in design, these are the critical loads. Hence a study of the factors of safety is essential. Furthermore, it would be more economical to design the tower without consideration of the brokenwire conditions and at the same time to prevent the total collapse of the tower in case a conductor breaks. A possible solution to this is to design the cross arms such that the cross arm turns away from the tower under the unbalanced tension in the event of breaking of a conductor. The rest of the tower structure would then be safe. Even material having

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The following abbreviations have been used in the names of journals.

JSD, ASCE - Journal of the Structural Division, American Society of Civil Engineers.

JEMD, ASCE - Journal of the Engineering Mechanics Division, American Society of Civil Engineers.

JPD, ASCE - Journal of the Power Division, American Society of Civil Engineers (Presently known as Journal of the Energy Division).

IJNME - International Journal for Numerical Methods in Engineering.

CS - Computers and Structures Journal

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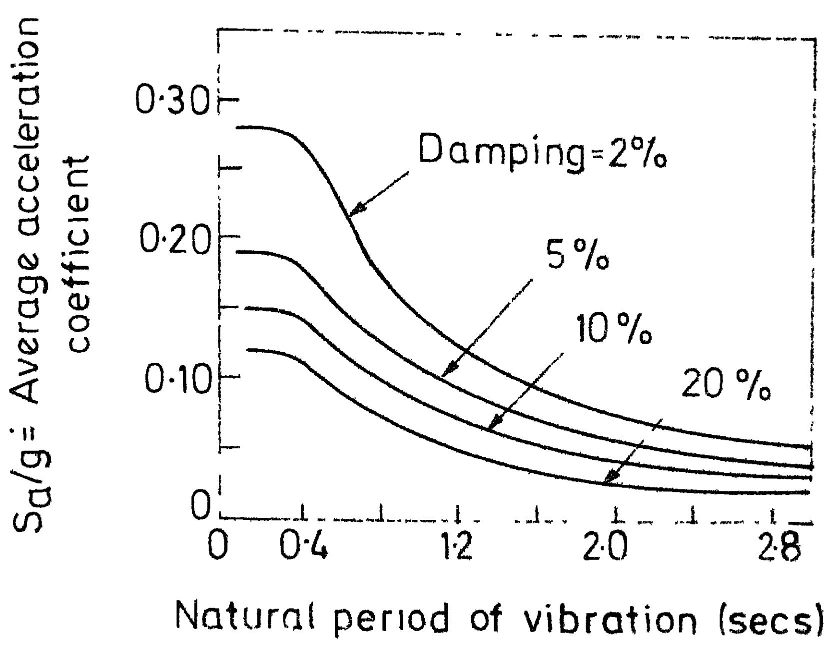

Fig. A1·1 Average acceleration spectra

APPENDIX 2

RATIONAL ANGLE SECTIONS

Introduction

Angle sections are extensively used as tension or compression members in industrial buildings, transmission towers, trusses etc. These widely used angles should have optimum dimensions so that their use as truss member will be economical[101,102].

Most of the angle sections that are listed in ISI Handbook[56] are found to be uneconomical. This observation is brought out in this study and the proposed rational sections are listed. Rational angles are derived with the aim that these angles are used mainly for truss members. The study is restricted to equal angle sections only.

Strength of Column Members

The maximum load that a column can carry without buckling is given by[99]

$$P = c\,\frac{\pi^2 EI}{l^2} \qquad (A2.1)$$

where c = constant which depends on the boundary conditions at the two ends (=1 for hinged ends, 4 for built-in ends),

E = modulus of Elasticity of the material,

I = least moment of inertia, i.e., minimum of the two principal moments of inertia,

and l = effective length of column.

From the above expression, it can be seen that the load carrying capacity of a column of given effective length is directly proportional to the least moment of inertia. That is, the value of I has to be maximized by properly dimensioning the cross section, to get maximum permissible compressive load. The relationship between moment of inertia and area of the section, A, is given by

$$I = A\,r^2 \qquad (A2.2)$$

where r = radius of gyration.

To achieve maximum economy, the section chosen should have minimum area and maximum radius of gyration. In other words maximize the least moment of inertia of the section for a given cross sectional area. This is possible by suitably dimensioning the section.

Sectional Properties of the Available Angles

To study the properties of the available angles, the least moment of inertia of these angles are listed against their increasing area of cross section ~~is~~ in Table A2.1. These values are also plotted in Fig.A2.1. From Table A2.1, it can

Table A2.1 Properties of the Indian Standard Angle Sections

Moment of Inertia for Increasing Area of Angles

Sl. No.	Area cm^2	I_{min} cm^4	Sl. No.	Area cm^2	I_{min} cm^4	Sl. No.	Area cm^2	I_{min} cm^4
1	1.12	0.2*	25	5.68	5.3	49	14.02	29.4
2	1.41	0.3*	26	5.75	7.7*	50	15.05	36.0
3	1.45	0.2	27	6.25	9.9*	51	15.39	58.4*
4	1.73	0.6*	28	6.26	7.0	52	17.02	78.2*
5	1.84	0.4	29	6.77	12.5*	53	17.03	51.6
6	2.03	0.9*	30	6.84	9.1	54	17.81	42.4
7	2.25	0.5	31	7.27	15.5*	55	19.03	71.8
8	2.26	0.7	32	7.44	11.7	56	20.19	60.9
9	2.34	1.4*	33	8.06	14.8	57	20.22	131.4*
10	2.64	2.0*	34	8.18	9.1	58	21.06	96.3
11	2.66	1.2	35	8.66	18.4*	59	22.59	84.7
12	2.77	0.9	36	8.96	11.9	60	25.02	113.8
13	2.95	2.8*	37	9.29	22.5*	61	25.06	162.1*
14	3.07	1.8	38	9.76	15.3	62	29.03	249.4*
15	3.27	1.5	39	10.02	11.2	63	29.82	191.8
16	3.47	2.6	40	10.47	32.0*	64	30.81	139.3
17	3.78	2.2	41	10.58	19.3	65	34.59	296.0*
18	3.86	1.7	42	11.00	14.6	66	36.81	235.0
19	3.88	3.6*	43	11.38	24.0	67	42.78	363.8*
20	4.28	3.2	44	11.67	44.5*	68	46.61	715.9*
21	4.47	2.6	45	12.00	18.8	69	50.79	429.5
22	4.79	4.5*	46	12.21	29.4	70	57.80	883.7*
23	5.07	3.8	47	13.02	23.7	71	68.81	1046.5*
24	5.27	5.9*	48	13.79	42.0	72	93.80	1411.6*

be observed that some of the sections (marked *) have special sectional properties, among the available sections. The least moment of inertia of these sections is increasing monotonically as the area increases. There are 29 sections out of the total 72, having such special property. Hence these sections will be preferred to others in the design of struts. For example, the section having area 2.95 cm^2 would be recommended instead of 3.86 cm^2 since the least moment of inertia of the latter is less than that of the former, i.e., load carrying capacity of the former is greater.

From Fig.A2.1, it can be observed that the above special sections fall far away on the left from the mean curve The equation of the mean curve is obtained by least square technique and the relationship between A and I is given by,

$$A = 2.48294\ (I)^{0.47276} \qquad (A2.3)$$

In an automated design, using the sections listed in Table A2.1, the sections picked up would be one of the angles marked * in Table A2.1, if the only consideration in the design is Euler buckling load. Other sections, if adopted will lead to uneconomical design.

Permissible Stresses in Angle Sections

From the dimensions of the 29 special sections (marked * in Table A2.1), it can be observed that the b/t

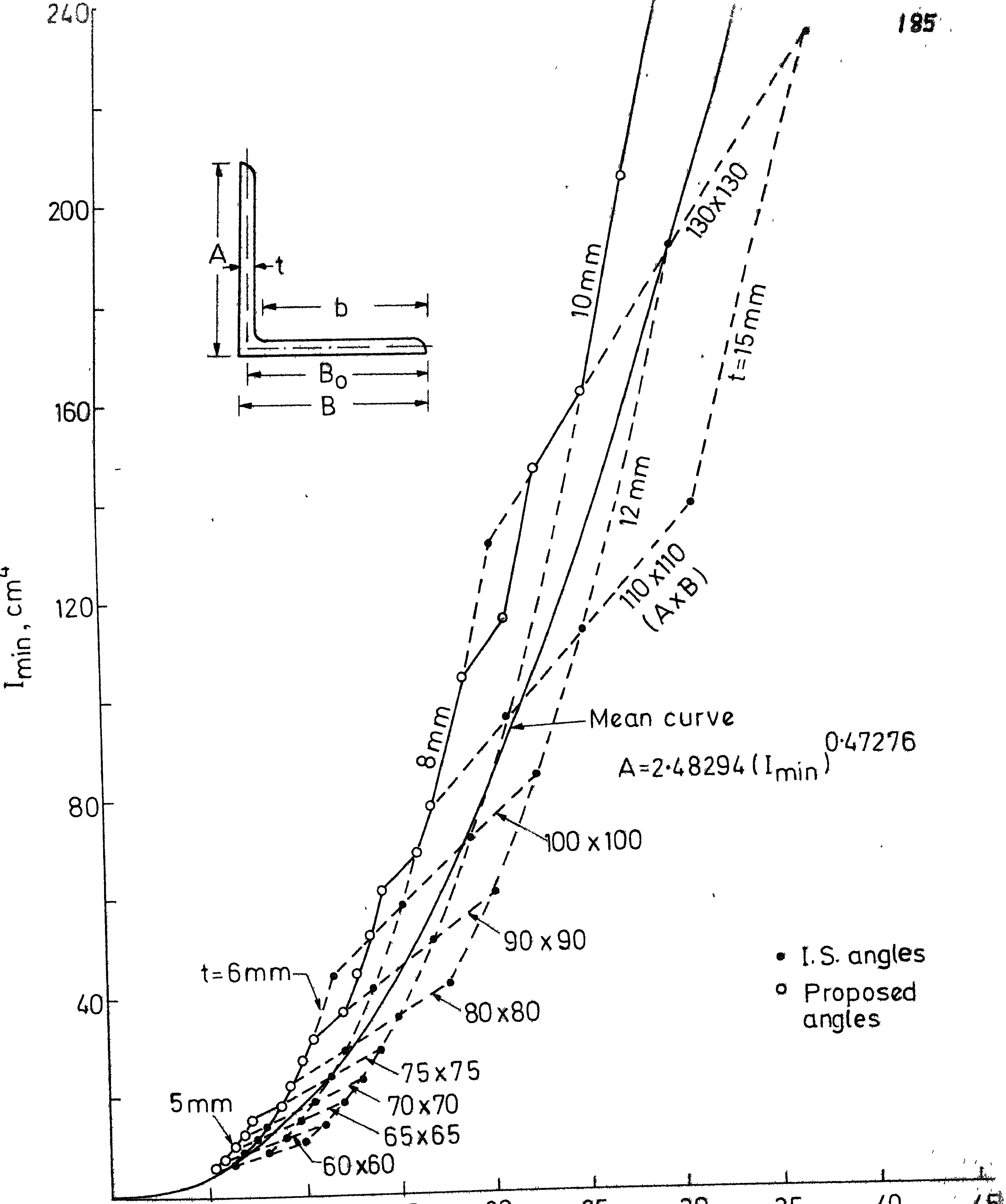

Fig.A2.1 Variation of moment of inertia with area

ratio (defined below) varies from 13 to 16 for most of these sections. In general the least moment of inertia can be increased infinitely by increasing the b/t ratio for a given value of area. But the allowable stress in such a section gets reduced so as to avoid local buckling of such thin flanges.

IS:802 (part I)-1973[55] stipulates reduction in permissible stress as the b/t ratio increases. For steel confirming to IS:226-1969[@] these formulae are given as follows:

(i) For the largest effective slenderness ratio $\left(\frac{KL}{r}\right) \leq 120$:

$$F_a = \left(1.0 - \frac{\left(\frac{KL}{r}\right)^2}{31200}\right) 2600 \text{ kgf/cm}^2 \quad \text{where } b/t \leq 13 \qquad (A2.4a)$$

$$F_a = \left(1.0 - \frac{\left(\frac{KL}{r}\right)^2}{31200}\right) \left(4680 - 160\frac{b}{t}\right) \text{ kgf/cm}^2 \quad 13 \leq \frac{b}{t} \leq 20 \qquad (A2.4b)$$

$$F_a = \left(1.0 - \frac{\left(\frac{KL}{r}\right)^2}{31200}\right) \frac{590000}{(b/t)^2} \text{ kgf/cm}^2 \quad \text{for } b/t > 20 \qquad (A2.4c)$$

(ii) For $\left(\frac{KL}{r}\right) > 120$

$$F_a = \frac{20{,}000{,}000}{\left(\frac{KL}{r}\right)^2} \text{ kgf/cm}^2 \qquad (A2.5)$$

where F_a = permissible stress in compression

K = fixity factor

L = length of the compression member

@ Specification for structural steel (standard quality) fourth revision.

$\frac{KL}{r}$ = largest effective slenderness ratio of any unbraced segment of the member

b = distance from the edge of fillet to the extreme fibre

and t = thickness of material.

When $(\frac{KL}{r}) > 120$, the permissible stress, Eq.(A2.5) is independent of (b/t) ratio and it is nothing but Euler buckling load, Eq. (A2.1) divided by area, i.e., when the member is slender (b/t) ratio can be large since the permissible load is Euler buckling load itself. In practice, slender members are adopted only for secondary bracings and for few primary diagonals. Hence, for these cases, any one of the nominal sections can be used.

Most of the leg members of the transmission towers and main members of the trusses have to be designed for heavy axial loads and also they are not slender. Since these members of the structure form the major portion of the materials consumed, they should be designed economically to gain overall reduction in the materials. Hence attention has to be focussed on the members having $(\frac{KL}{r})$ ratio less than 120.

Rational Angles

With the aim of reducing material consumption, one would naturally like to take the angles having minimum area.

Area of the section will be least when the allowable stress of the section is maximum. Furthermore, as discussed in the earlier sections, the dimensions of the angle should be chosen with the aim of having minimum area and maximum strength.

Let B_o be the distance from the edge of one flange of equal angle (Fig. A2.1) to the centre line of the other and k is the b/t ratio.

$$k = \frac{b}{t} \simeq \frac{B_o - t/2}{t} \; ; \; t = \frac{2B_o}{(2k+1)}$$

Area of cross section, $A = 2B_o t = \dfrac{4B_o^2}{(2k+1)}$ (A2.6a)

and $$\frac{dA}{dk} = - \frac{8B_o^2}{(2k+1)^2} \qquad \text{(A2.6b)}$$

From the ISI Handbook[56], it is seen that the radius of gyration is almost constant for different thickness, t, of angles of a given flange width B and hence it is independent of thickness t. This can also be observed from Fig.A2.1, where the I_{min} versus A is almost a straight line for a fixed flange width. So the quantity $\left[1.0 - \frac{1}{31200}\left(\frac{KL}{r}\right)^2\right]$ in Eqs.(A2.4) is a constant for a fixed value of B.

The ultimate load carrying capacity, P is given by

$$P = AF_a = \frac{4B_o^2}{(2k+1)} F_a \qquad \text{(A2.7)}$$

and $$\frac{dP}{dk} = \frac{dA}{dk} F_a + A \frac{dF_a}{dk}$$

Substituting the expressions from Eq. (A2.4)

$$\frac{dP}{dk} = \frac{dA}{dk} 2600 \ C, \quad k \leq 13 \qquad \text{(A2.8a)}$$

$$= \frac{dA}{dk} \left[2600-160 \ (k-13) \right] C - A \times 160 \ C; 13 < k < 20$$

$$= \frac{dA}{dk} 2600 \ C - \left[\frac{dA}{dk}(k-13) + A \right] 160 \ C \qquad \text{(A2.8b)}$$

$$= \frac{dA}{dk} \frac{590000}{k^2} C - A \frac{590000 \times 2}{k^3} C \qquad k \geq 20 \qquad \text{(A2.8c)}$$

where $C = \left[1.0 - \frac{1}{31200} \left(\frac{KL}{r}\right)^2 \right]$

The above equations reveal that for unit change in area of the section the corresponding change in capacity of section is maximum in the sections for which $k \leq 13$. Hence sections having $\frac{b}{t} \leq 13$ are well utilized than those having $\frac{b}{t} > 13$. In otherwords, the sections for which the allowable stress is maximum are rational. In the earlier sections it was observed that $\frac{b}{t}$ ratio can be chosen as high as possible to get better sectional properties. These observations point to the fact that the sections should be dimensioned in such a way that $\frac{b}{t} \simeq 13$ to get optimum use.

Recommended Angle Sections

Based on the observations in the previous sections

the proposed rational angles are listed in Table A2.2. As far as possible the proposed sections have k value equal to 13. The sectional properties of these angles are plotted in Fig. A2.1 along with the mean curve for the presently available angles.

When the sections are chosen to have rational dimensions, the number of sections obtained are a few for the standard width of angles available in ISI Handbook. So additional sections having width in between those available are also recommended. While doing so, only those angle sections having monotonic increase in sectional properties are presented. There are 48 such sections, listed in Table A2.2. These are closely spaced. Hence these angles when used in an automated design would produce rational design in comparison to the presently available sections.

The equation of the mean curve for the recommended angles in terms of A and I_{min} is given by

$$A = 2.275(I_{min})^{0.4654} \qquad (A2.9)$$

The deviation of the sectional properties of the proposed angles from the above mean curve (not drawn in Fig.A2.1) is very small. Hence, in case a continuous function in terms of area and moment of inertia is required, in any optimization study, Eq. (A2.9) could be used without much of error.

Table A2.2 Proposed Equal Angle Sections

Sl.No.	Size AxBxt mm	Area cm^2	I_{min} cm^4
1	20x20x3	1.12	0.2
2	20x20x4	1.45	0.2
3	25x25x3	1.41	0.3
4	25x25x4	1.84	0.4
5	30x30x3	1.73	0.6
6	30x30x4	2.26	0.7
7	35x35x3	2.03	0.9
8	35x35x4	2.66	1.2
9	40x40x3	2.34	1.4
10	40x40x4	3.07	1.8
11	45x45x3	2.64	2.0
12	45x45x4	3.47	2.6
13	50x50x4	3.88	3.6
14	55x55x4	4.26	4.9
15	55x55x5	5.27	5.9
16	60x60x5	5.75	7.7
17	65x65x5	6.25	9.9
18	70x70x5	6.77	12.5
19	75x75x5	7.27	15.5
20	75x75x6	8.66	18.4
21	80x80x6	9.29	22.5
22	85x85x6	9.87	27.6
23	90x90x6	10.47	32.0
24	90x90x7	12.14	37.2
25	95x95x7	12.84	44.8
26	100x100x7	13.54	52.6

Table A2.2 contd....

Sl.No.	Size AxBxt mm	Area cm^2	I_{min} cm^4
27	105x105x7	14.25	61.3
28	105x105x8	16.20	69.2
29	110x110x8	17.02	78.2
30	120x120x8	18.62	104.2
31	120x120x9	20.85	116.6
32	130x130x9	22.65	146.0
33	130x130x10	25.06	162.1
34	140x140x10	27.05	206.2
35	150x150x10	29.03	249.4
36	150x150x11	31.82	277.6
37	160x160x11	34.08	340.0
38	160x160x12	37.05	368.2
39	170x170x12	39.45	444.4
40	180x180x12	41.83	528.8
41	180x180x13	45.18	569.0
42	190x190x13	47.78	672.9
43	190x190x14	51.31	720.1
44	200x200x14	54.09	841.9
45	200x200x15	57.80	883.7
46	200x200x16	61.49	951.2
47	200x200x17	65.16	1005.0
48	200x200x18	68.81	1046.5

The use of the proposed angles in transmission line tower design has been described in Section 4.5. The optimum weights of tower obtained with the proposed angles are compared to the one obtained by using the available sections, Table 4.6. From the results of the optimization study, it is confirmed that the recommended sections (Table A2.2) could be used economically in the construction of any truss-like structures.

APPENDIX 3

RAYLEIGH-RITZ METHOD

The subspace iteration method uses the Rayleigh-Ritz principles and the Eq. (5.10) to Eq. (5.14) are nothing but the steps involved in Rayleigh-Ritz method. Hence the Rayleigh-Ritz method is briefly described below:

Let V_n denote the n-dimensional space in which the operators K and M are defined. The Rayleigh minimum principle states that

$$\lambda_1 = \omega_1^2 = \min.\ \rho(\vec{\phi}) \qquad (A3.1)$$

in which the minimum is taken over all vectors $\vec{\phi}$ in V_n and $\rho(\vec{\phi})$ is the Rayleigh quotient defined as

$$\rho(\vec{\phi}) = \frac{\vec{\phi}^T K \vec{\phi}}{\vec{\phi}^T M \vec{\phi}} > 0 \qquad (A3.2)$$

The numerator in the above equation is twice the maximum strain energy for a given $\vec{\phi}$ (displacement vector in structural application) and the denominator is twice the maximum kinetic energy of the structure, divided by ω^2. The minimum of $\rho(\vec{\phi})$ can be obtained by any one of the methods of unconstrained minimization[58].

In the Ritz analysis all the vectors $\vec{\phi}$ in a q-dimensional subspace V_q of V_n are considered. A typical

element in the subspace is given by linear combinations of Ritz basis vectors $\vec{x}_i$,

$$\vec{\phi} = \sum_{i=1}^{q} a_i \vec{x}_i \qquad (A3.3)$$

in which a_i = the Ritz coordinates. Substituting Eq. (A3.3) in Eq. (A3.2),

$$\rho(\vec{\phi}) = \frac{\sum_{j=1}^{q}\sum_{i=1}^{q} a_j a_i \tilde{k}_{ij}}{\sum_{j=1}^{q}\sum_{i=1}^{q} a_j a_i \tilde{m}_{ij}} \qquad (A3.4)$$

is obtained with $\tilde{k}_{ij} = \vec{x}_i^T K \vec{x}_j$

and $\tilde{m}_{ij} = \vec{x}_i^T M \vec{x}_j$ (A3.5)

The necessary condition for a minimum of $\rho(\vec{\phi})$ is $\frac{\partial \rho(\vec{\phi})}{\partial a_i} = 0$, $i=1,2, \ldots q$, because a_i are the only variables. This gives

$$\tilde{K} \vec{a} = \lambda \tilde{M} \vec{a} \qquad (A3.6)$$

where $\vec{a}$ is a vector listing the Ritz coordinates; $\tilde{K}$ and $\tilde{M}$ are matrices of size q x q with typical elements defined in Eq. (A3.5).

The solution to Eq. (A3.6) yields q values $\bar{\lambda}_1, \bar{\lambda}_2, \ldots \bar{\lambda}_q$ and the corresponding vectors $\bar{\vec{\phi}}_1, \ldots\ldots\ldots \bar{\vec{\phi}}_q$, which are obtained using Eq. (A3.3). The values $\bar{\lambda}_i$, $i=1,\ldots,q$ are upper bound[97,98] approximations to the exact eigenvalues of the original problem, Eq. (5.4), i.e.,

$$\omega_1^2 \leq \lambda_1 \; ; \; \omega_2^2 \leq \lambda_2 ; \; \ldots . \; ; \; \omega_q^2 \leq \lambda_q \qquad \text{(A3.7)}$$

The error in the solution depends on the Ritz basis vectors chosen because the approximate eigenvectors $\vec{\bar{\phi}}_1, \ldots, \vec{\bar{\phi}}_q$ are the elements of the subspace.

APPENDIX 4

SOLUTION OF GENERALIZED SYMMETRIC EIGENVALUE PROBLEM

The subspace iteration method requires the solution of the generalized symmetric eigenvalue problem[100], Eq. (5.13). Since the size of the matrices involved are only qxq and also it is necessary to calculate all the q eigenpairs, Jacobi method recommends itself[84].

The method of Jacobi to solve the generalized eigen-problem

$$(A - \lambda B)\,\vec{g} = \vec{0} \qquad (A4.1)$$

is presented, here, with matrices A and B symmetric and known. In the presentation below, A will be same as K_{k+1} and B is same as M_{k+1} in Eq. (5.13). The requirement in this method is that B should be positive definite. In the present case, $B = M_{k+1} = \bar{X}^T_{k+1}\, M\, \bar{X}_{k+1}$ is nonsingular, since it is quadratic.

First, the positive definite matrix B of the generalized symmetric eigenvalue problem $(A - \lambda B)\,\vec{g} = \vec{0}$ is to be transformed by Jacobi's method to a diagonal form D, using the orthogonal matrix E:

$$D = E^T\, B\, E \qquad (A4.2)$$

The generalized eigenproblem is transformed with the help of matrix E in that, on one hand, the identity matrix is introduced in the form EE^T, and on the other, the eigenvalue equation is multiplied from the left by E^T:

$$E^T (A - \lambda B) EE^T \vec{g} = \vec{0} \qquad (A4.3)$$

Eq. (A4.3) is rewritten, using Eq. (A4.2) as

$$(E^T A E - \lambda D) E^T \vec{g} = \vec{0} \qquad (A4.4)$$

The generalized eigenvalue problem is hereby very nearly converted into the special one. In place of the identity matrix I, Eq. (A4.4) has a diagonal matrix D with only positive elements which are equal to the eigenvalues of B. The diagonal matrix $D^{\frac{1}{2}}$, consisting of the square roots of the positive diagonal elements of D, exists, as does its inverse $D^{-\frac{1}{2}}$. Thus Eq. (A4.4) can be further transformed into

$$D^{-\frac{1}{2}}(E^T A E - \lambda D) D^{-\frac{1}{2}} D^{\frac{1}{2}} (E^T \vec{g}) = \vec{0}$$

or $$(D^{-\frac{1}{2}} E^T A E D^{-\frac{1}{2}} - \lambda I)(D^{\frac{1}{2}} E^T \vec{g}) = \vec{0} \qquad (A4.5)$$

The matrix $T = D^{-\frac{1}{2}} E^T A E D^{-\frac{1}{2}}$ (A4.6)

must be symmetric since A is symmetric, E is orthogonal, and $D^{-\frac{1}{2}}$ is a diagonal matrix. Through

$$\vec{h} = D^{\frac{1}{2}} E^T \vec{g} \qquad (A4.7)$$

the generalized eigenvalue problem is reduced to a special one with symmetrix matrix T, i.e.,

$$(T - \lambda I) \vec{h} = \vec{0} \tag{A4.8}$$

The eigenvalues λ of T agree with that of the generalized eigenvalue problem $(A - \lambda B) \vec{g} = \vec{0}$, and the interrelationship of the eigenvectors of $\vec{h}$ and $\vec{g}$ are given by (A4.7). The special eigenvalue problem, Eq.(A4.8) can be again solved by Jacobi's method. Here we encounter the transformation matrix V containing the eigenvectors of T in its columns. Through Eq. (A4.8) the matrix G of the eigenvectors of the problem Eq. (A4.1) can be obtained.

The algorithm for finding the eigenvalues λ_i and eigenvectors $\vec{g}_i$ of a generalized symmetrix eigenvalue problem $(A- \lambda_i B) \vec{g}_i = \vec{0}$ consists of the following steps:

$$\begin{aligned}
&(a) \quad E^T B E = D \qquad \text{(Jacobi)} \\
&(b) \quad T = D^{-\frac{1}{2}} E^T A E D^{-\frac{1}{2}} \\
&(c) \quad V^T T V = D_1 \qquad \text{(Jacobi)} \\
&(d) \quad G = E D^{-\frac{1}{2}} V
\end{aligned} \tag{A4.9}$$

Eigenvalues λ_i are the diagonal elements of D_1 and the corresponding eigenvectors are the appropriate columns of matrix G. Matrix G is itself not orthogonal, instead we have $G^T B G = I$, where the generalized metric is expressed.

The method of Jacobi, that is required in (A4.9) is well documented in literature[84,90,96].

APPENDIX 5

COMPUTER PROGRAM

```
      DIMENSION X(12),S(12,12),Z(12),Y(12),SD(12)
      ITER=1
      CALL DATAIN(X,N)
      F1=F(X)
      F1=F(X)
      WRITE(3,15) F1
   15 FORMAT('  STARTING FUNC.VALUE=',F10.3)
      IU=0
      DO 20 I=1,N
      DO 20 J=1,N
      S(I,J)=0.0
   20 IF(I.EQ.J) S(I,J)=1.0
      EP=0.01
      DO 23 I=1,N
   23 SD(I)=S(I,N)
   24 DO 25 J=1,N
   25 Y(J)=X(J)+EP*SD(J)
      CLESS=0.0
C     USE THIS FY (* REDUCING THE NO. OF FUNC.  EVALUATION
      FR=F(Y)
      FL=FR+10.0
      IF(FR.LT.F1) GO TO 50
      DO 30 J=1,N
      SD(J)=-SD(J)
   30 Y(J)=X(J)+EP*SD(J)
      FL=F(Y)
      IF(FL.LT.F1) GO TO 50
      CLESS=1.0
      IF(FL.LE.FR) GO TO 50
      DO 45 J=1,N
   45 SD(J)=-SD(J)
   50 WRITE(3,55) (SD(I),I=1,N)
   55 FORMAT('  S=',12F10.4/)
   56 CALL QUAD(X,SD,N,ALFA,F1)
      IF(ALFA.GT.0.0) GO TO 57
      IF(IU.EQ.(N+1)) GO TO 13
      IF(CLESS.GT.1.0)    GO TO 61
      ALFA=EP
      F1=FR
      IF(FR.GT.FL)    F1=FL
   57 DO 60 J=1,N
   60 X(J)=X(J)+ALFA*SD(J)
   61 ITER=ITER+1
      IF(ITER.GT.2) GO TO 63
      DO 62 I=1,
   62 Z(I)=X(I)
   63 WRITE(3,64) (X(I),I=1, )
   64 FORMAT('  X=',12F10.2)
       WRITE(3,65) F1,ITER
   65 FORMAT( 60X,'FUNC. VALUE=',F10.3,' AT THE ITERATION  NO.=',I3/)
   66 IU=IU+1
      IF(IU.EQ.1) GO TO 71
      IUP=IU-1
      DO 67 I=1,
   67 S(I,IUP)=SD(I)
      IF(IU.LE.N)  GO TO 71
       DO 68 I=1,
   68 WRITE(3,69) (S(I,J),J=1,N)
   69 FORMAT(12F10.4//)
      GO TO 80
   71 DO 70 I=1,
   70 SD(I)=S(I,IU)
      GO TO 24
   80 IF(IU.GE.(N+2)) GO TO 100
      DO 90 I=1,N
   90 SD(I)=X(I)-Z(I)
      WRITE(3,95) (SD(I),I=1, )
   95 FORMAT(5X,121('-')/'  PATTERN DIRECTION  '/12F10.5//)
C     CHECK  FOR  OPTIMUM
      ITER=1
      DO 96 I=1,
      IF(ABS(SD(I)).GT.0.001) GO TO 56
   96 CONTINUE
      GO TO 130
  100 N1=N-1
```

```
      DO 110 J=1,N1
      DO 110 I=1,N
  110 S(I,J)=S(I,J+1)
      DO 120 I=1,N
      Z(I)=X(I)
  120 S(I,N)=SD(I)
      IU=0
      GO TO 65
  130 STOP
      END
C
      SUBROUTINE QUAD(X,S,N,OS,FY)
C
C
      DIMENSION  X(12),Y(12),S(12)
      EPS=0.001
      A=0.0
      AT=0.0
      T=0.3
      IFIT=1
      FA=FY
      DO 10 I=1,N
   10 Y(I)=X(I)+ T*S(I)
      FY=F(Y)
      IF(FY.GT.FA) GO TO 20
   14 FB=FY
      B=AT+T
      DO 15 I=1,N
   15 Y(I)=Y(I)+T*S(I)
      FY=F(Y)
      IF(FY.GT.FB) GO TO 18
      AT=AT+T
      A=B
      FA=FB
      GO TO 14
   18 FC=FY
      C=AT+2.*T
      GO TO 30
   20 FC=FY
      C=T
      IF(T.LT.0.02) GO TO 120
      T=T/2.
      B=T
      DO 21 I=1,N
   21 Y(I)=X(I)+T*S(I)
      FY=F(Y)
      IF(FY.LT.FA) GO TO 29
      GO TO 20
   29 FB=FY
   30 ALFAS=(4.*FB-3.*FA-FC)*T/(4.*FB-2.*(FA+FC))
      OS=AT+ALFAS
      DO 31 I=1,N
   31 Y(I)=X(I)+OS*S(I)
      FY=F(Y)
      HALFAS=FA+ALFAS*(4.*FB-3.*FA-FC)/(2.*T)+ALFAS**2*(FC+FA-2.*FB)/
     1 (2.*T*T)
      ER=ABS((HALFAS-FY)/FY)
      WRITE(3,35) A,FA,B,FB,C,FC,AT,ALFAS,OS,FY,HALFAS,ER
   35 FORMAT(12F10.4/)
      IF(ER.LE.EPS) GO TO 100
      IFIT=IFIT+1
      IF(IFIT.GT. 5) GO TO 100
      IF(B.GT.OS) GO TO 50
      IF(FB.LT.FY) GO TO 40
      AT=B
      A=B
      FA=FB
      B=OS
      FB=FY
      GO TO 30
   40 C=OS
      FC=FY
      GO TO 30
   50 IF(FB.GT.FY) GO TO 60
      A=OS
```

```
      FA=FY
       AT=OS
      GO TO 30
  60  C=B
      FC=FB
      B=OS
      FB=FY
      GO TO 30
 100  IF(FB.GE.FY) GO TO 105
      FY=FB
      OS=B
 105  WRITE(3,110) OS,IFIT
 110  FORMAT(' OPTIMAL STEP=',F10.5,' NO. OF TTPS=',I5/)
      GO TO 140
 120  WRITE(3,130)
 130  FORMAT(50X,' FUNCTION DOES NOT DECREASE ALONG THIS DIRECTION'/)
      OS=0.0
      FY=FA
 140  RETURN
      END
C
      FUNCTION F(DV)
      DIMENSION DV(10)
      CALL
      CALL
      F=WEIGHT
      RETURN
      END
C
      SUBROUTINE DATAIN(I ,  )
      COMMON   ,CK(261,30),U(261,6),F(261,6),A (261),  C,IPASS, OF,
     1IFICE ,FSTR (261,3),  (261,10)
      COMMON/  / X(98),Y(98),Z(98),IP(316),I (316), (316),
     1 SMD(6,6),  (12), S,   ,   ,  AL
      COMMON /FIXCRO/ T,BODY T,ISTART,CDV,KXTYPE,   DT (12)
      COMMON/FIXDAT/    ,     S,  C,   ,LJ(7),F (7,27),IPRINT,AC
      COMMON/GROUPS/     (5 ),     ( ),ITYPE(50)
      COMMON/EVAL/ EIG  (10),      ( ),LOAD  (10),  L
      DIMENSION  DV(12)
C         OPEN(UNIT=1,DEVICE='DSK',FILE='MEDIUM.DAT',     =     )
C
       READ 10,    TS,   S,  C,  C,  J,ISTART,CDV,KXTYPE,IPRINT,
     1 IEIGEN
      WRITE(3,10)   TS,   BS,  C,  C,  J,ISTART,       ,IPRINT,
     1 IEIGEN
  10  FORMAT(10I5)
       READ 15,        T,BODYT
      WRITE(3,15) T,BODYT
  15  FORMAT(2F10.4)
C               1Y
C               1-----X
C              /
C             Z
      IEND=ISTART-1
      N=0
C     PROVIDE NODAL  DATA  FOR  EACH NODE IN CASE OF  
C    IT=0 FOR
C    IT=1 FOR
  16  READ 20,  I,IT,X(J),Y(J),Z(J)
      WRITE(3,20)J,IT,X(J),Y(J),Z(J)
  20  FORMAT(2I5,3F10.4)
      I=1
  21  N=N+1
      IF(N.EQ. ) GO TO 30
      I=I+1
       Y(N)=Y(N-1)
       IF(ITYPE.  .1) GO TO 25
      X(N)=-X(N-1)
      Z(N)=Z(N-1)
      IF(I.NE.3) GO TO 28
      X(N)=-X( )
      Z(N)=-Z( )
      GO TO 28
  25  X(N)=Z(N-1)
      Z(N)=X(N-1)
```

```
      IF(I.EQ.3) Z(N)=-Z(N)
   28 IF(N.LT.J) GO TO 21
   30 ITYPE=IT
      IF(N.LT.IEND ) GO TO 16
      MS2=MS/2
      N=NJNTS*MS2
      NJ=MS2*NLC
   31 DO 32 I=1,NLJ
       READ   33, LJ(I),(FJ(I,J),J=1,NJ)
   32 WRITE(3,33) LJ(I),(FJ(I,J),J=1,NJ)
   33 FORMAT(I8,9F8.2)
      ULOAD=0.0
   34 DO 35 I=1,N
      DO 35 J=1,NLC
   35 F(I,J)=0.0
      DO 36 M=1,NLJ
      J=LJ(M)*MS2-MS2-1
      DO 36 L=1,NLC
      I=J+1
      DO 36 K=1,MS2
      I=I+1
      N1=(L-1)*MS2+K
   36 F(I,L)=FJ(M,N1)
      IF( [illegible] ) GO TO 140
      DO 37 I=1,N
      DO 37 J=1,NLC
   37 FSTORE(I,J)=F(I,J)
C
       M=0
C     IT=1 FOR LEG [illegible]
C     IT=2 FOR K BRACINGS
C     IT=3 FOR X BRACINGS
C     IT=4 FOR TE SIO  AND [illegible]
C     IT=5 FOR THRUSTO[illegible] BRACES [illegible]
C     IT=6 FOR X BRACING IN [illegible]
C      PROVIDE [illegible] DATA FOR LAST [illegible]
   39 READ    40, J, IT,IP(J),IQ(J),A(J)
      WRITE(3,38) J,IT,IP(J),IQ(J),A(J)
   38 FORMAT(4I5,F10.2)
   40 FORMAT(4I3,F8.2)
      I=1
   42 M=M+1
      IF(M.EQ.J)GO TO 51
      I=I+1
      IP(M)=IP(M-1)+1
      IQ(M)=IQ(M-1)+1
      GO TO (50,43,44,47,46,41),ITYPE
   41 IQ(M)=IQ(M)-2
      GO TO 50
   43 IF(I.EQ.5) GO TO 48
      IF(I.EQ.8) GO TO 49
      GO TO 50
   44 IF(I.EQ.5) GO TO 48
      IF(I.EQ.8) GO TO 49
      GO TO 47
   46 IF(I.NE.5) GO TO 47
      IP(M)=IP(M)-1
      IQ(M)=IQ(M)-1
      GO TO 50
   47 IF(I.EQ.4) IQ(M)=IQ(M)-4
      GO TO 50
   48 IP(M)=IP(M)-3
      IQ(M)=IQ(M)-1
      IF(ITYPE.EQ.2) IQ(M)=IQ(M)-3
       GO TO 50
   49 IP(M)=IP(M)-1
   50 A(M)=A(M-1)
      IF(M.LT.J) GO TO 42
   51 ITYPE=IT
      IF(M.LT.MEMBS) GO TO 39
   59  READ 60,(JST(J),J=1,NC)
      WRITE(3,60) (JST(J),J=1,NC)
   60 FORMAT(16I5)
       MAXDIF=0
      DO 90 M=1,MEMBS
```

```
      IDIF=IABS(IO(M)-IP(M))
   90 IF(IDIF.GT.MAXDIF) MAXDIF=IDIF
      IBAND=NS2*(MAXDIF+1)
      NDOF=N-NBC
      WRITE(3,100) NDOF,IBAND
  100 FORMAT(/15X,'  STRUCTURE IDEALIZED AS TRUSS',/'    NO. OF DEGREES
     1N DISPLACEMENTS=',I5,/'            HALF BAND WIDTH=',I5/)
       READ 110 ,(DV(I),I=1,NDV)
      WRITE(3,110) (DV(I),I=1,NDV)
  110 FORMAT(8F10.4)
       READ 120 ,(PANDIV(I),I=1,NDV)
      WRITE(3,120) (PANDIV(I),I=1,NDV)
  120 FORMAT(12F5.0)
      DO 130 I=1,NDV
      IF(PANDIV(I).GE.2.) GO TO 130
      M=27+2*I
       MSDOM(M-1)=IFIX(PANDIV(I))
      MSDOM(M)=MSDOM(M-1)
  130 CONTINUE
      NVAR=NDV
      NCOUNT=0
      IF(IETGEN.EQ.0) GO TO 150
      DO 135 I=1,NLC
  135 ETGVAL(I)=0.0
C
       ULOAD=1.0
      NLJ=5
      NLC=6
      NJ=NS2*NLC
      DO 136 I=1,NLJ
       READ 137, LI(I),(PJ(I,J),J=1, J)
  136 WRITE(3,137) LI(I),(PJ(I,J),J=1,  J)
  137 FORMAT(I4,9F8.2/(8X,9F8.2))
      GO TO 34
C
C     IETGEN.LE.3    WIND GOOD DYNAMIC
C     IETGEN .GT.6    EARTHQUAKE  LOAD
  140 IF(IETGEN.GE.11) GO TO 150
      READ 10, SOL,(LOAD(I),I=1,NLC)
      WRITE(3,10)SOL,(LOAD(I),I=1,NLC)
  150 RETURN
      END
C
      SUBROUTINE  XYZ(    ,  EIGHT)
      COMMON/KONAP/ X(   ),Z(   ),Y(   ),I (316),IO(316), (316),
     1 SKD(6,6), IA(12),  ,   ,  32,  ,
      COMMON /FIXGEO/  ,   ,  ,IS   ,   ,  ,KXTYPE,      (  )
      DIMENSION XS( ),ZS( ),X ( ),  ( ),  (12)
      DATA XS,ZS/-1.0,1.0,1.0,-1.0,1.0,1.0,-1.0,-1./
      DATA Y ,Z /0.0,1.0,0.0,-1.0,1.0,0.0,-1.0,0./
      B=DV(NDV)
      C=(B-T)/(2.0*BODYHT)
      XX=T/2.
      ZZ=XX
      IE=ISTART+3
      II=1
      DO 10 I=ISTART,IE
      X(I)=XS(II)*XX
      Z(I)=ZS(II)*ZZ
      IF(KXTYPE.EQ.1) GO TO 9
      X(I+4)=X (II)*XX
      Z(I+4)=Z (II)*ZZ
      Y(I+4)=BODYHT
    9 II=II+1
   10 Y(I)=BODYHT
      HT=0.0
      NP=NDV-1
       DO 15 I=1,NP
   15 HT=HT+DV(I)
C     THE  BOTTOM  MOST PANEL IS DEPENDENT ON OTHERS. HENCE IT IS NOT A
C      DESIGN VARIABLE
      DV(NDV )=BODYHT-HT
      HT=BODYHT
      DO 25 KP=1,NDV
      HT=HT-DV(KP)
```

```
      XX=XX+C*DV(KP)
      ZZ=XX
      DO 20 I=1,4
      J=ISTART+4*KP*KXTYPE+I-1
      X(J)=XS(I) *XX
      Z(J)=ZS( I)* ZZ
      IF(KXTYPE.EQ.1) GO TO 20
      X(J+4)=XM(I)*XX
      Z(J+4)=ZM(I)*ZZ
      Y(J+4)=HT
   20 Y(J)=H T
   25 CONTINUE
C      C OMPUTATION OF THE LENGTH OF SEC.BRACES -- X AND K BRACES (=1,
      VOL=0.0
      DO 60 I=1,NDV
      II=(I-1)*4*KXTYPE+ISTART
      J=II+4*KXTYPE
      ISUB=1
      X  OVE=(X(I )-X(J))/PANDIV(I)
      Y  OVE=  (I)/PANDIV(I)
      XLEG=X(J)
      YLEG=0.0
      XINMOV=-X(J)/P   IV(I)
      XINNER=X(J)
   30 GO TO (31,31,32,31,32,31,  ),ISUB
   31 XLEG=XLEG+X  OVE
      YLEG=YLEG+Y  OVE
   32 IF(KXTYPE.EQ.1) GO TO 40
      GO TO (35, 6,35,46,35,46),ISUB
   35 XINNER=XLEG+X
      YINNER=YLEG
      GO TO 46
   40 IF(PANDIV(I). . . ) GO TO 42
      GO TO (41,46,43,44,46,46),ISUB
   41 XINNER=X(J)/2.0
      YINNER=X(J)/(X(J)+X(II))*DV(I)*0.5
      GO TO 46
   42 ISUB=6
   43 XINNER=0.0
      YINNER=X(J)/(X(J)+X(II))*DV(I)
      GO TO 46
   44 XLEG=XLEG-X  OVE
      YLEG=YLEG-Y  OVE
       ISUB=ISUB+1
      XINNER=X(II)/2.0
      YINNER=(X(J)+XIN  )/(X(J)+X(II))*DV(I)
   46 SECLEN=SQRT((   LEG-X    ER)**2+(YLEG-YI   ER)**2)
C     WRITE(6,47) I,   ,S   ,
   47 FORMAT('          =',I3,'     SEC. BRACE    =',I3,'
     1 BRACE=',  .2)
      IF(SECLEN.GT.2.47) GO TO 50
      VOL=VOL+SECLE  *8.0*3.47*100.0
      GO TO 55
   50 AREA=3.9
      CALL DESIG (AREA,SECLEN,-( .01, . , , ,   ,   )
      VOL=VOL+SECLEN*AREA*8.  *100.
   55 ISUB=ISUB+1
      IF((X(II)-XLEG).GT.0.001) GO TO 60
      GO TO 30
   60 CONTINUE
      WEIGHT=VOL
      WRITE(3,70) WEIGHT
   70 FORMAT('   WEIGHT OF SECONDARY STRUC=',  .2)
      DV(NDV)=S
      RETURN
      END
C
      SUBROUTINE ANALYS( EIGHT)
      COMMON N,GK(261,30),U(261,6),F(261,6),A (261),NC,
     1 IETGEN,FSTORE(261,3),TEMP(261,3)
      COMMON/KTDAT/ X(98),Y(98),Z(98),IP(316),IQ(316),(316),
     1SMD(6,6), JK(12), S,RO , S2,  ,E,
      COMMON/FIXDAT/ ,    ,   ,  AC,  ,LJ(7),FI(7,27),
      COMMON/GROUPS/    G(50),  SPO (50), TYPE(50)
      COMMON/EVAL/ EIGVAL(10), (10),  (10),
```

```
      IF(IEIGEN.EQ.0) GO TO 192
C
      DO 191 I=1,NDOF
      AM(I)=AM(I)/0.981
      DO 191 J=1,IBAND
  191  TEMP(I,J)=GK(I,J)
C
C      DYNAMIC  RESPONSE  CALCULATION
      IF(EIGVAL(1).GT.0.0) GO TO 193
      CALL BAND
  193 I=1
      CALL DYMRES(NMODE)
       GO TO 197
  192 CALL BAND
  197 IF(IPRINT.EQ.0) GO TO 196
      DO 194 N1=1,NJNTS
      K=N1*MS2-MS2+1
      L=N1*MS2
  194 WRITE(3,195) N1,((U(I,J),I=K,L),J=1,NC)
  195 FORMAT(I5,)F12.1)
C    CALCULATION OF BAR  FORCES
  196 IS=1
      IG=1
      IF=NMTNG(IG)
  200 FMIN=0.
      FMAX=0.
      ITYPE=MTYPE(IG)
      DO 270 M=IS,IF
      NB=IB(M)
      NE=IQ(M)
      J3=3*NB
      J2=J3-1
      J1=J3-2
      K3=3*NE
      K2=K3-1
      K1=K3-2
      HX=X(NE)-X(NB)
      HY=Y(NE)-Y(NB)
       V=Z(NE)-Z(NB)
      AL=SQRT(HX*HX+HY*HY+V*V)
      CX=HX/AL
      CY=HY/AL
      CZ=V/AL
      IF(IEIGEN.EQ.0)   GO TO 225
      DO 220 I=1,NC
      DFF=0.0
       DO 210 MO=1,NMODE
      UX=U(K1,MO)-U(J1,MO)
      UY=U(K2,MO)-U(J2,MO)
      UZ=U(K3,MO)-U(J3,MO)
      DEFMOD=(UX*CX +UY*CY +UZ*CZ)*[illegible](I,MO)
C     GF(MO)=UGF(I)*DEFMOD*E*A(M)/(AL*1000.0)
      DFF=DFF +ABS(DEF  [illegible])
  210 CONTINUE
      AXIF(I)=DFF*UGF(I)*E*A(M)/(AL*1000.0)
C     WRITE(6,195)M,([illegible], [illegible]=1,NMODE), AXIF(I)
      IF(AXIF(I).GT.[illegible]) [illegible]=AXIF(I)
  220 CONTINUE
      IF(ITYPE.EQ.1) GO TO 271
      IF(M.GE.(IS+1)) GO TO 271
       GO TO 270
  225 DO 230 I=1,NC
      UX=U(K1,I)-U(J1,I)
      UY=U(K2,I)-U(J2,I)
      UZ=U(K3,I)-U(J3,I)
      AXIF(I)=(UX*CX+UY*CY+UZ*CZ)*E*A(M)/(AL*100.0)
      AXIF(I)=AXIF(I)*UGF(I)/1000.
      IF(AXIF(I).LT.FMIN) FMIN=AXIF(I)
  230 IF(AXIF(I).GT.FMAX) FMAX=AXIF(I)
  265 IF(IPRINT.EQ.0)   GO TO 270
      WRITE(3,280) M,IB(M),IQ(M),A(M),AL,(AXIF(I),I=1,NC)
  270 CONTINUE
  271 IF(IEIGEN.GT.0)   FMIN=-FMAX
      AA=A(IS)
C
```

```
      NSD=NSDOM(IG)
      CALL DESIGN(AA,AL,FMIN,FMAX,ITYPE,NSD,ACF,ATF)
C     WRITE(6,280) IG,IS,IE,AL,A(IS),FMIN,FMAX,AA,ACF,ATF
      DO 275 M=IS,IE
  275 A(M)=AA
      WEIGHT=WEIGHT+RO*FLOAT(MING(IG))*AA*AL*100.0
      CAREA(IG,NCOUNT)=AA
      IG=IG+1
      IS=IE+1
      IE=IE+MMING(IG)
      IF(IS.LE.NMEMBS) GO TO 200
  280 FORMAT(3I5,2F6.2,5F10.3)
      IF(IPRINT.EQ.0) GO TO 300
       DO 285 I=1,NJNTS
       L=I*3
      K=L-2
  285 WRITE(3,54) I,((F(J,M),J=K,L),M=1,NLC)
  300 WRITE(3,175) WEIGHT
      IF(NCOUNT.LT.20) GO TO 330
      IG=IG-1
      DO 310 I=1,IG
  310 WRITE(3,320) I,(CAREA(I,J),J=1,20)
  320 FORMAT(I5,20F6.2)
      NCOUNT=0
  330 IF(IEIGEN.GT.0)  GO TO 340
      DO 335 I=1,N
      DO 335 J=1,NLC
  335 F(I,J)=[illegible]
  340 RETURN
      END
C
      SUBROUTINE STIFF
      COMMON/KHMAT2/ X(98),Y(98),Z(98),IP(316),IQ(316),A(316),
     1 SMD(6,6),JM(12),IS,NOU,IS2,NC,AL
      NB=IP(1)
      NE=IQ(1)
      HX=X(NE)-X(NB)
      HY=Y(NE)-Y(NB)
      V=Z(NE)-Z(NB)
      AL=SQRT(HX*HX+HY*HY+V*V)
      CX=HX/AL
      CY=HY/AL
      CZ=V/AL
      AL=AL*100.0
      C=E*A(1)/AL
      IF(ABS(CX).LT.0.05.AND.ABS(CZ).LT.0.05) GO TO 30
    5 SMD(1,1)=C*CX*CX
      SMD(1,2)=C*CX*CY
      SMD(1,3)=C*CX*CZ
      SMD(2,2)=C*CY*CY
      SMD(2,3)=C*CY*CZ
      SMD(3,3)=C*CZ*CZ
      DO 10 I=4,6
      K=I-3
      DO 10 J=I,6
       SMD(I,J)=SMD(K,J-3)
   10 SMD(K,J)=-SMD(I,J)
      SMD(2,4)=SMD(1,5)
      SMD(3,4)=SMD(1,6)
      SMD(3,5)=SMD(2,6)
      DO 20 I=2,6
      II=I-1
      DO 20 J=1,II
   20 SMD(I,J)=SMD(J,I)
      GO TO 50
   30 DO 40 I=1,6
      DO 40 J=1,6
   40 SMD(I,J)=0.0
      SMD(2,2)=C
      SMD(5,5)=C
      SMD(2,5)=-C
      SMD(5,2)=-C
   50 RETURN
      END
C
```

```
      SUBROUTINE BAND
      COMMON NT,A(261,30),X(261,6),Y(261,6),AM(261),NLC,NB,NDOF,[illegible]
     1N   ,ESTORE(261,3),TEMP(261,30)
      N=NDOF
      M=N-1
      DO 100  I=1,M
      DO 90  J=2,NB
      IJ=I+J-1
      IF(IJ-N)10,10,100
 10   IF(A(I,J))15,90,15
 15   AL=A(I,J)/A(I,1)
      NJ=NB-J+1
      DO 90 K=1, NJ
      KJ=K+J-1
 80   A(IJ,K)=A(IJ,K)-AL*A(I,KJ)
      DO 85 L=1,NLC
 85   Y(IJ,L)=Y(IJ,L)-AL*Y(I,L)
 90   CONTINUE
100   CONTINUE
      DO 110  I=1,NLC
110   X(N,I)=Y(N,I)/A(N,1)
      DO 155  I=1,M
      K=N-I
      DO 145  J=2,NB
      KJ=K+J-1
      IF(KJ-N)140,140,150
140   DO 145  L=1,NLC
145   Y(K,L)=Y(K,L)-A(K,J)*X(KJ,L)
150   DO 155  L=1,NLC
155   X(K,L)=Y(K,L)/A(K,1)
      RETURN
      END

      BLOCK  DATA
      COMMON/[illegible]/ [illegible](99),[illegible](99),[illegible](99),T(99),[illegible]X(45),[illegible](45),
     1 AMLRR(4)  ,[illegible](99)
      COMMON/[illegible]/X(98),Y(98),Z(98),IP(316),[illegible](316),A(316),[illegible],
     1 JB(12),[illegible],[illegible],[illegible],[illegible],AG
      COMMON/GROUPS/ [illegible](50),[illegible](50),MTYPE(50)

      DATA AREA/3.47,3.88,4.28,4.79,5.07,5.27,5.68,5.73,6.25,6.26,
     1 6.77,6.84,7.27,7.44,8.06,8.18,8.56,8.46,[illegible],[illegible],10.02,10.47
     2 10.58,[illegible],[illegible],[illegible],12.00,12.21,13.02,13.79,14.02,15.05,[illegible]
     3 ,17.02,[illegible],[illegible],19.03,20.10,20.22,21.[illegible],[illegible],25.05,[illegible]
     4 59.03,[illegible],[illegible],34.59,36.81,42.78,[illegible],[illegible],[illegible],[illegible]
     5 ,2.86,[illegible],[illegible],4.16,3.47,4.76,5.26,[illegible],[illegible],2,6.25,
     6 6.27,[illegible],[illegible],[illegible],7.45,[illegible],8.58,8.55,[illegible],[illegible],10.14,
     7 10.52,11.37,[illegible],11.56,[illegible],12.57,12.86,[illegible],[illegible],15.38,[illegible]
     8 ,16.50,[illegible],[illegible],17.42,18.02,19.55,21.[illegible],[illegible],[illegible],25.[illegible]/

      DATA [illegible] /0.37,[illegible],[illegible],[illegible],[illegible],1.03,[illegible],[illegible],[illegible],1.36,
     1 1.12,[illegible],[illegible],[illegible],[illegible],[illegible],1.15,[illegible],[illegible],[illegible],[illegible],
     2 1.15,1.15,[illegible],[illegible],[illegible],[illegible],1.75,[illegible],[illegible],[illegible],[illegible],[illegible],
     3 1.54,[illegible],[illegible],[illegible],[illegible],[illegible],2.13,[illegible],[illegible],[illegible],[illegible],
     4 2.92,3.[illegible],[illegible],[illegible],[illegible],[illegible],[illegible],[illegible],[illegible],[illegible],[illegible],
     5 0.96,0.76,[illegible],[illegible],[illegible],1.07,0.45,[illegible],[illegible],[illegible],[illegible],[illegible],
     6 1.06,1.39,[illegible],[illegible],[illegible],1.28,1.16,1.42,[illegible],[illegible],[illegible],1.27,
     7 1.51,1.38,[illegible],1.37,[illegible],[illegible],1.[illegible],[illegible],[illegible],1.50,[illegible],1.53/

      DATA [illegible]/[illegible],[illegible],[illegible],[illegible],[illegible],55.,50.,[illegible],65.,55.,[illegible],60.,75.,65.,
     1 70.,55.,75.,[illegible],[illegible],65.,55.,90.,70.,65.,75.,[illegible],65.,[illegible],70.,
     2 90.,75.,[illegible],[illegible],110.,[illegible],80.,100.,[illegible],[illegible],110.,100.,[illegible],[illegible],
     3 150.,130.,[illegible],160.,130.,150.,200.,[illegible],[illegible],[illegible],200.,[illegible],[illegible],
     4 45.,50.,[illegible],50.,[illegible],65.,70.,[illegible],75.,[illegible],[illegible],70.,75.,[illegible],[illegible],[illegible],
     5 70.,90.,75.,100.,[illegible],100.,75.,[illegible],[illegible],[illegible],[illegible],[illegible],[illegible],[illegible],
     690.,125.,100.,[illegible],[illegible],125.,150.,125.,[illegible],[illegible],[illegible],[illegible],[illegible]/

      DATA T/4.,[illegible],5.,[illegible],6.,[illegible],6.,[illegible],5.,[illegible],5.,[illegible],5.,6.,6.,8.,[illegible],[illegible],
     1 8.,10.,6.,[illegible],[illegible],[illegible],[illegible],10.,[illegible],10.,[illegible],[illegible],[illegible],[illegible],12.,[illegible],
     2 12.,8.,10.,15.,12.,10.,10.,12.,15.,12.,15.,15.,[illegible],[illegible],[illegible],[illegible],
     3 25.,4.,4.,5.,[illegible],5.,6.,5.,5.,5.,[illegible],[illegible],[illegible],[illegible],[illegible],[illegible],[illegible],
     4 6.,8.,6.,8.,6.,10.,8.,10.,6.,10.,[illegible],6.,[illegible],10.,[illegible],10.,[illegible],[illegible],[illegible],
     5 8.,10.,12.,10.,10.,12.,12./

      DATA RXX/1.41,1.58,1.4[illegible],1.57,1.39,1.56,1.3[illegible],2.15,2.22,[illegible],2.3[illegible],
```

```
1 2.04,2.55,2.21,2.37,1.80,2.50,2.02,2.19,2.20,2.35,2.1 ,2.52,3.15,
2 2.16,2.84,2.33,3.01,2.49,3.16,3.97,3.14,2.91,4.00,3.14,3.12,
3 2.79,3.96,4.83,3.07,3.19,3.94,1.81,3.91,4.71/

  DATA CW/30.,30.,30.,30.,30.,30.,40.,45.,45.,40.,50.,45.,50.,45.,
1 50.,40.,50.,45.,45.,60.,50.,65.,50.,75.,45.,60.,50.,75.,50.,55.,
2 95.,75.,60.,75.,65.,75.,60.,95.,75.,75.,75.,95.,75.,75.,75./

  DATA R1/5.5,6.0,5.5,6.0,5.5,6.5,6.0,6.5,6.5,6.5,7.0,6.5,7.0,6.5,
1 7.0,6.5,7.0,6.5,8.0,6.5,6.5,8.5,7.0,6.5,7.5,3.5,6.5,3.0,7.5,
2 8.5,7.0,8.0,8.5,10.0,8.5,8.0,8.5,8.5,10.0,10.0,8.5,10.0,10.0,
3 12.0,10.0,15.0,12.0,10.0,12.0,15.0,12.0,15.0,15.0,15.0,5.0,
4 5.5,5.0,5.5,5.0,5.5,6.0,6.0,6.5,5.0,6.5,5.0,7.0,6.5,6.5,5.0,
5 7.0,6.0,6.5,7.5,6.5,8.0,7.0,8.5,6.5,7.5,6.5,9.0,7.0,8.0,9.0,8.5,
6 7.5,9.0,8.0,8.5,7.5,9.0,10.0,9.0,8.5,9.0,10.0,9.0,10.0/

  DATA AMLRR/150.,200.,250.,200./
  DATA  S,F,E  /6,0.20470E 07,0.78300E-02/
  DATA  N  G/   3,7*8,6*4,4,8,4,4,4,3,4,3,4,8,4,8,4,8,4,8,4,8,4,8,4,
1 8,0/
  DATA  ISHOP/7*2,7*1,7*2,6*1,22*2,0/
  DATA  ITYPE/7*4,7*1,7*2,6*4,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,
1 1,2,1,2,0/
  END

  SUBROUTINE DESIGN(AL,ST,FC,FP,ITYPE,N30,ACF,ATF)
  COMMON/A   /  A  (99),   (99),  (99),T(99),RXX(35),  (45),
1  AMLRR(4)  ,R1(99)
  IF(ITYPE.EQ.2) GO TO 5
  ISTART=1
  IEND=54
  GO TO 6
5 ISTART=55
  IEND=99
6 ALRL=AMLRR(ITYPE)
  ALR1=0.
  CORR=0.
  FC=ABS(FC)
  RAREA=FC/2.0
  RREQD=AL*100./(ALRL*FLOAT(N30))
9 DO 10 I=ISTART,IEND
  IF(RAREA.GT.AREA(I)) GO TO 10
  IF(RREQD.LE.R1(I)) GO TO20
10 CONTINUE
20 ACF=0.
  ATF=0.
  IF(ITYPE.EQ.3) GO TO 600
25 IF(FC.EQ.0.0) GO TO 200

  ALR=100.*AL/(R1(I)*FLOAT(N30))
  IF(ITYPE.EQ.2) ALR1=75.0*AL/RXX(I-54)
  IF(ALR1.GT.ALR) ALR=ALR1
  IF(ALR.GT.ALRL) GO TO 300
  IF(ALR.GT.120.) GO TO 30
  WT=( A(I)- (I)-R1(I) )/T(I)
  IF(WT.GT.13.) GO TO 40
  STRES=2.6-ALR**2/12000.
  GO TO 70
30 STRESS=20000.0/ALR**2
  GO TO 70
40 IF(WT.GT.20.) GO TO 50
  FCR=4600.0-130.*WT
  GO TO 60
50 FCR=590000./WT**2
60 STRESS=(1.0-ALR**2/2*S00.)*FCR/1000.
70 IF(STRESS.LE.2.0) GO TO 80
  IF(CORR.GT.0.0) GO TO 80
  RAREA=FC/2.
  CORR=2.0
  GO TO 9
80 ACF=STRESS*AREA(I)
100 IF(ACF.LT.FC) GO TO 300
200 IF(ITYPE.EQ.2) GO TO 250
  ATF=2.6*(AREA(I)-1.43*T(I))
  IF(ATF.GE.FT)  GO TO 600
```

```
      GO TO 300
  250 AA=T(I)*(CH(I-51)-21.5)/100.
      B=(W(I)-T(I))*T(I)/100.
      AK=1.0/(1.0+0.35*B/AA)
      ATF=2.6*(AA+B*AK)
      IF(ATF.GE.FT)  GO TO 600
  300 I=I+1
      IF(I.LE.NEND) GO TO 25
      I=NEND
  600 A=AREA(I)
      RETURN
      END

      SUBROUTINE DYNRES(NMODE)
      COMMON N,GK(261,30),  X(261,6),F(261,6),AM(261),NLC,IHALF, NOR,
     1 IETGEN,FSTORE(261,3),TEMP(261,30)
      COMMON/EVAL/ EIGVAL(10),DLEMAX(10),LOADNO(10),NL
      COMMON/NORMOD/ AMXX(10),GF(10),TMAX(5,10)

      CALL SSPACE
      NMODE=6
      CALL DYNAMC(NMODE)

      DO 5 I=1,N
      DO 5 J=1,NLC
    5 F(I,J)=FSTORE(I,J)

      DO 10 J=1,NMODE
      AMXX(J)=0.0
      DO 10 I=1,NOR
   10 AMXX(J)=AMXX(J)+AM(I)*X(I,J)**2
      WRITE(3,55)    (AMXX(I),I=1,NMODE)
C     DLEMAX IS STORED IN AM
      DO 20 I=1,NOR
   20 AM(I)=1.0
      DO 80 L=1,NLC
      DO 50 M=1,NMODE
      DO 30 J=1,NL
      I=LOADNO(J)
   30 AM(I)=DLEMAX(J)
      GF(M)=0.0
      DO 40 I=1,NOR
   40 GF(M)=GF(M)+X(I,M)*F(I,L)*AM(I)
   50 TMAX(L,M)=GF(M)/(AMXX(M)*EIGVAL(M))
      WRITE(3,55) (TMAX(L,M),M=1,NMODE)
   55 FORMAT(10F10.4)
   80 CONTINUE
      RETURN
      END
C
      SUBROUTINE SSPACE
      COMMON N,GK(261,30),X(261,6),F(261,6),AM(261),NLC, [illegible], NOR,IETGEN,
     1 M ,FSTORE(261,3),TEMP(261,30)
      COMMON/VECVAL/CKR(10,10),AMR(10,10),H(10,10),TE (10)
      COMMON/EVAL/ EIGVAL(10),DLEMAX(10),LOADNO(10), NL
      NLC=6
      WRITE(3,12)   (EIGVAL(I),I=1,NLC)   [illegible]
C   THE MASS IS [illegible] DEP CHANGE FOR [illegible]
    5 DO 10 I=1,NOR
      DO 10 J=1,NLC
   10 F(I,J)=AM(I)*X(I,J)
      WRITE(3,12) (EIGVAL(I),I=1,NLC)
   12 FORMAT(10E10.5)
      DO 15 I=1,NOR
      DO 15 J=1,M
   15  GK(I,J)=TEMP(I,J)
C   ITERATE FROM F(K-1) TO F(K)
      CALL BAND
C    FIND THE PROJECTIONS OF THE OPERATORS [illegible]
      DO 20 I=1,NOR
      DO 20 J=1,M
   20  GK(I,J)=TEMP(I,J)
C     MULTIPLY K X(K) AND M X(K) AND THEN PREMULTIPLY
```

```
      DO 30 I=1,NDOF
      N1=I-NB+1
      IF(N1.LE.0) N1=1
      DO 30 J=1,NLC
      F(I,J)=0.0
      DO 25 K=N1,NDOF
      KI=I-K+1
      IF(KI.LE.0) GO TO 26
   25 F(I,J)=F(I,J) +GK(K,KI)*X(K,J)
      GO TO 30
   26 K=I+1
      DO 28 KI=2,NB
       F(I,J)=F(I,J)+GK(I,KI)*X(K,J)
      K=K+1
      IF(K.GT.NDOF) GO TO 30
   28 CONTINUE
   30 CONTINUE
      DO 35 I=1,NDOF
      DO 35 J=1,NLC
      GK(I,J)=F(I,J)
   35 F(I,J)=AM(I)*X(I,J)
      DO 40 I=1,NLC
      DO 40 J=1,NLC
      GKR(I,J)=0.
      AMR(I,J)=0.
      DO 40 K=1,NDOF
      GKR(I,J)=GKR(I,J)+X(K,I)*GK(K,J)
   40 AMR(I,J)=AMR(I,J)+X(K,I)*F(K,J)
      EPS=0.0001
      CALL GENEIG(NLC,EPS)
C   FIND AN IMPROVED APP. TO EIGENVECTORS
       DO 60 I=1,NDOF
       DO 50 J=1,NLC
       TEM(J)=0.0
       DO 50 K=1,NLC
   50  TEM(J)=TEM(J)+X(I,K)*U(K,J)
       DO 60 J=1,NLC
   60  X(I,J)=TEM(J)
C     CHECK FOR CONVERGENCE
        DO 70 I=1,NLC
      DIFF=ABS(AMR(I,I)-EIGVAL(I))/AMR(I,I)
      IF(DIFF.GE.0.00001) GO TO 75
   70 CONTINUE
      DIFF=0.
   75 DO 80 I=1,NLC
   80 EIGVAL(I)=AMR(I,I)
      IF(DIFF.GT.0.0) GO TO 5
      WRITE(3,90)
   90 FORMAT('     EIGEN VALUES CONVERGED ')
      WRITE(3,12) (EIGVAL(I),I=1,NLC)
      DO 95 I=1,NDOF
95       WRITE(3,12)   (X(I,J),J=1,NLC)
           CALL RCHECK
       NLC=3
       RETURN
       END
C
       SUBROUTINE GENEIG(N,EPS)
C
       COMMON/ VECVAL/ A(10,10),B(10,10),U(10,10),EIGV(10)
       DIMENSION BS(10,10)
       DO 10 I=1,N
       DO 10 J=1,N
       BS(I,J)=B(I,J)
       U(I,J)=0.
   10  IF(I.EQ.J) U(I,J)=1.
       ICOMP=0
C     START JACOBI ITERATIONS ON A MATRIX
       WRITE(3,25)
       IPRINT=0
       NLSS1=N-1
   15  SS=0.
       DO 20 I=1,NLSS1
       IPLS1=I+1
       DO 20 J=IPLS1,N
```

```
      IF(ABS(B(I,J)).LT.0.00001) GO TO 20
      SS=SS+B(I,J)**2
      IF(SS.GT.1000.) GO TO 22
   20 CONTINUE
C  TEST WHETHER SS SUFFICIENTLY SMALL
   22 WRITE(3,26)   SS
   25 FORMAT('  SUM OF OFF DIAGONAL ELEMENTS SQUARED IS ...')
      FORMAT(55X,E15.8)
      IF(2.*SS-EPS**2) 100,30,30
C   NO,IT IS NOT , KEEP COMPUTING
   30 DO 85 IP=1,NLSS1
      J1=IP+1
      DO 85 IQ=J1,N
      IF(ABS(B(IP,IQ)).LT.0.000001) GO TO 85
C     APPLY P-Q  ROTATION
   40 THETA=0.5*(B(IQ,IQ)-B(IP,IP))/B(IP,IQ)
      IF(THETA) 60,50,60
   50 T=1.
      GO TO 70
   60 POS=1.
      IF(T...) POS=-1.
      T=1.0/(...+...(1.0+THETA**2))
   70 C=1.0/SQRT(1.+T*T)
      S=C*T

C    COMPUTE  ROTATION

      H=C*C*B(IP,IP)-...(IP,IQ)+S*S*B(IQ,IQ)
      G=S*S*B(IP,IP) ...    (I ,I )+C*C*B(I ,I )
      B(IP,IQ)=C*S*( (IP,IP)- (I ,I ))+(C*C-S*S)*B(I ,IQ)
      B(IP,IP)=H
      B(IQ,IQ)=G
      IF(IP-1)73,73,71
   71 IPLSS1=IP-1
      DO 72 J=1,IPLSS1
      H=C*B(J,IP)-S*B(J,IQ)
      B(J,IQ)=S*B(J,IP)+C*B(J,IQ)
   72 B(J,IP)=H
   73 IF(IQ-IP-1) 80,7 ,74
   74 IPPLS1=IP+1
      IQLSS1=IQ-1
      DO 75 J=IPPLS1,IQLSS1
      H=C*B(IP,J)-S*B(J,IQ)
      B(J,IQ)=S*B(IP,J)+C*B(J,IQ)
   75 B(IP,J)=H
   76 IF(N-IQ) 80,80,77
   77 IQPLS1=IQ+1
      DO 78 J=IQPLS1,N
      H=C*B(IP,J)-S*B(IQ,J)
      B(IQ,J)=S*B(IP,J)+C*B(IQ,J)
   78 B(IP,J)=H
C
C     COMPUTE UK= U(K-1) ...
C
   80 DO 82 I=1,
      H=U(I,IP)*C-U(I,I )*S
      U(I,IQ)=U(I,IP)*S+U(I,IQ)*C
   82 U(I,IP)=H
   85 CONTINUE
      GO TO 15
C     COMPUTE C=D- UP A U D-
  100 IF(ICOMP.GT.0) GO TO 200
      DO 110 I=1,N
  110 B(I,I)=1.0/SQRT(B(I,I))
      DO 120 J=1,
      DO 120 I=1,
  120 U(I,J)=U(I,J)*B(J,J)
      DO 130 I=1,N
      DO 130 J=1,
      B(I,J)=0.
      DO 130 K=1,
  130 B(I,J)=B(I,J)+A(I,K)*U(K,J)
      DO 150 J=1,
      DO 140 I=1,N
      TEM(I)=0.
```

```
      DO 140 K=1,N
 140  TEM(I)=TEM(I)+A(K,I)*B(K,J)
      DO 145 I=1,N
 145  B(I,J)=TEM(I)
 150  CONTINUE
      ICOMP=1
      GO TO 15
C     CHECK   WHETHER   UT B U=I
 200  IF(IPRINT.EQ.0)  GO TO 250
      DO 210 I=1,N
      DO 210 J=1,N
      A(I,J)=0.
      DO 210 K=1,N
 210  A(I,J)=A(I,J)+BS(I,K)*U(K,J)
      DO 220 I=1,N
      DO 220 J=1,N
      BS(I,J)=0.
      DO 220 K=1,N
 220  BS(I,J)=BS(I,J)+U(K,I)*A(K,J)
 230  FORMAT(10E13.5)
      DO 240 I=1,N
 240  WRITE(3,230) (BS(I,J),J=1,N)
 250  RETURN
      END

      SUBROUTINE [illegible]
     [illegible]
      COMMON [illegible],A(261,[illegible]),[illegible](261,[illegible]), [illegible]
     1 [illegible]
      COMMON/EVAL/ EIG[illegible](10), [illegible]
      C=0.99
    4 DO 1000 I1=1,[illegible]
      EL=C*EIGVAL(I1)
      DO 6 I=1,NDOF
      DO 6 J=1,NB
    6 A(I,J)=TEMP(I,J)
      DO 7 I=1,NDOF
    7 A(I,1)=A(I,1)-EL*[illegible](I)
C     ELIMINATE THE LOWER TRIANGULAR ELEMENTS
      N=NDOF
      M=N-1
      DO 100 I=1,M
      DO 90 J=2,NB
      IJ=I+J-1
      IF(IJ-N) 10,10,100
   10 IF(A(I,J))15,90,15
   15 AL=A(I,J)/A(I,1)
      NBJ=NB-J+1
      DO 80 K=1,NBJ
      KJ=K+J-1
   80 A(IJ,K)=A(IJ,K)-AL*A(I,KJ)
   90 CONTINUE
  100 CONTINUE
      WRITE(3,105) EL
  105 FORMAT(//' SHIFT= ',E10.5/)
      WRITE(3,110) (A(I,1),I=1,[illegible])
  110 FORMAT(5X,12E10.3)
 1000 CONTINUE
      IF(C.GT.1.0) GO TO 1200
      C=1.01
      GO TO 4
 1200 STOP
      END
C
      SUBROUTINE DY[illegible]
      COMMON/EVAL/ EIGVAL(10),[illegible](10),[illegible](10),[illegible]
      DIMENSION [illegible](12),TYPEF(11),[illegible](12)
      DATA [illegible]/[illegible],3,[illegible],1.15,1.3[illegible],2.[illegible],2.65,4.,4.25,4.50,[illegible]/
      DATA TYPEF/1.[illegible],1.0,-1.0,1.0,1.0,-1.0,[illegible],1.0,-1.[illegible]/
      DATA PAI,P,PRC[illegible]/3.1415927, 0.5 ,0.05 /
      DO 110 KK=1,[illegible]
      W=SQRT(EIGVAL(KK)*1000.)
      FREQ(KK)=W
      BETA=PRC[illegible]*[illegible]
    5 W2=W*W
```

```
      WD=SQRT(W2-BETA**2)
      WD2=WD*WD
      WDT=WD*TINTER(2)
      EBT=EXP(-BETA*TINTER(2))
      S=SIN(WDT)
      C=COS(WDT)
      T=TINTER(2)
      YO=( T-2.*BETA/W2 +(2.*BETA*C+(BETA**2-WD2)*S/WD)*EBT/W2)*P/T
      DYO=(1.-EBT*(C+BETA*S/WD)   )*P/T
      F1=P
      DLF=YO
      DO 100 J=2,11
      VARIF=YDEF(J)
      TDIFF=TINTER(J+1)-TINTER(J)
      IF(VARIF.[illegible])) GO TO 6
      EBT=EXP([illegible]IFF)
      [illegible]=TD[illegible]
      S=SIN([illegible])
      C=COS([illegible]T)
      YT=P+[illegible]*( [illegible])*C+ (DYO+BETA*(YO-P))*S/WD))
      DYT=EBT*( [illegible]*C-(DYO*BETA+W2*(YO-P))*S/WD))
      DDYO=-[illegible]*( [illegible]*(YO-P)+2.*DYO*BETA)*C+(DYO*(WD2-BETA**2)-W2*BETA*
     1 (YO-P))*S/WD)
      YO=YT
      DYO=DYT
      GO TO 100
    6 A=YO-F1+(1.[illegible])*2.[illegible]/(TDIFF+ [illegible])
      B=(DYO+BET[illegible](YO-[illegible]))/ [illegible]*( [illegible])+( [illegible]*2- [illegible])/( [illegible]*TDIFF)
      C1=-(A*(WD2-[illegible]
      C2=B*(WD2-[illegible]**2)-[illegible]
      WDT=ATAN2(C1,C2)
      ITER=0
      IF(WDT.GT.0.0) GO TO 11
      WDT=2.*PAI+WDT
   11 IF(WDT.LT.PAI) GO TO 12
      WDT=WDT-PAI
   12 T=WDT/WD
      TS=T
   13 IF(T.LT.TDIFF) GO TO 14
      IF(T.LT.0.0) GO TO 26
      T=TDIFF
   14 S=SIN(WD*T)
      C=COS(WD*T)
      EBT=EXP(-BETA*T)
      YT=F1+VARIF*(1.0-P)*(T-2.*BETA/W2)/[illegible]
      DYTOLD=DYT
      DYT=VARIF*(1.0-P)/TDIFF+[illegible]*( [illegible])*C+(B*(WD2-BETA**2)-2.+
      DDYT=-EBT*((A*(WD2-BETA**2)+2.[illegible])*S)
     1   BETA*A*WD)*S)
      T1=T+TINTER(J)
C     PRINT 15, T1,YT,DYT,DDYT
   15 FORMAT(4F10.5)
      IF(YT.GT.DLF) DLF=YT
      IF(T.GE.TDIFF) GO TO 30
      IF(ITER-1) 16,11,25
   16 ADDT=2.0
      ITER=1
      IF(DDYO.GT.0.0) GO TO 17
      IF(DYO.GT.0.0) GO TO 18
      TS=TS+PAI/WD
   27 T=TS
      GO TO 13
   17 T=TS
      GO TO 19
   18 T=0.0
      DYT=DYO
      ADDT=1.
   19 IF(DYT.LT.0.0) GO TO 20
      T=T+0.1*PAI/WD
      GO TO 13
   20 IF(DYTOLD.LT.0.0) GO TO 26
      T=T-0.05*PAI/WD
      ITER=2
      GO TO 13
   25 T=T-DYT/DDYT
```

```
      IF(ABS(DYT).GE.0.   [illegible]
   26 TS=TS+ADDT*PAI/[illegible]
      ADDT=0.
      ITER=1
      GO TO 27
   30 YO=YT
      DYO=DYT
      DDYO=DDYT
      F1=F1+VARIF*(1.0-P)
       PRINT 90
   90 FORMAT(//)
  100 CONTINUE
      DLFMAX(KK)=DLF
  110 CONTINUE
      WRITE(3,120)  (FUFJ(I),I=1,MMODE)
      WRITE(3,120) (DLFMAX(I),I=1,MMODE)
  120 FORMAT(10F10.5)
      RETURN
      END
```